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ANALYSIS OF ARGUS BOMB DEBRIS

Mathematical Model

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FOREWORD

This is the final report of the work by Lockheed Missiles & Space Company performed under Air Force Contract AF 29(601)-4419 with the Air Force Special Weapons Center, 1 June 1961 – 21 December 1962. It is presented in two volumes: AFSWC TDR-62-127, Vol I, Behavior of the Debris and Related Phenomena, classified Secret-Restricted Data; and AFSWC TDR-62-127, Vol II, Mathematical Model, unclassified.

The Report is identified for Lockheed purposes as LMSC-B007043, Code No. 2-96-62-1.

ABSTRACT

This volume contains a detailed description of the analytical methods used in constructing the Monte Carlo model and code. The various mechanisms contributing to the energy losses of heavy particles moving through the atmosphere are reviewed, and it is concluded that for the cases of interest here only the elastic collisions need to be taken into account. An outline is given of a statistical treatment of inelastic collisions together with a derivation of the corresponding probability distribution. The probability distribution for the beta decays of fission particles is also derived, and lastly, the logical construction of the code for the calculation of a history of a given particle is described in detail.

PUBLICATION REVIEW

This report has been reviewed and is approved.

DONALD I. PRICKETT
Colonel USAF
Director, Research Directorate

for JOHN J. DISHUCK
Colonel USAF
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INTRODUCTION

This report is a continuation of a study carried out by the Theoretical Physics Group at Lockheed on the motion of the bomb debris. This volume describes the mathematical background of the Monte Carlo code used in this study and the code itself.

We would like to acknowledge the contributions of A. J. Cook to the formulation and execution of the Monte Carlo calculations.

SUMMARY

In any Monte Carlo model it is advantageous to limit the number of microscopic processes which have to be treated statistically. In the model of behavior of fission particles in the atmosphere, the most important processes are perhaps the collision of these particles with the atmospheric atoms. This is the subject of Section 1, Energy Loss of Fission Particles Released at High Altitudes, where the contributions of the elastic and inelastic collision to the slowing down of a heavy particle are briefly examined. It is found that approximate calculations of such contributions are valid for velocities up to about 5×10^7 cm/sec, and that, for the same velocities, a good first approximation consists of neglecting the energy loses due to inelastic collision.

With this assumption, it is possible to integrate out the contributions of the elastic collisions, and only the inelastic collisions have to be treated statistically. The appropriate probability distribution is derived in Section 2., Inelastic Collisions of Fission Particles in the Atmosphere. This distribution depends only on the cross section for the collision and the initial altitude, and is strongly determined by the analytical assumption made on the behavior of the density with altitude. The different forms of the distribution in various atmospheres are briefly examined. The choice of the final form is governed by simplicity of formulation and the necessity for rapidly developing a working code.

In Section 3, Beta Decay of Fission Fragments, a probability distribution for the beta decay of a typical fission particle is derived. The assumption is made that the activity of a collection of fission particles is adequately given by the Way-Wigner law, and a prescription for obtaining a good approximation to this law is given.

In Section 4 , IBM 7090Code for the Monte Carlo Model, the detailed steps in the calculation of a history of a fragment are described. It must be pointed out that all the steps shown and the associated numerical procedures were chosen primarily to develop in a reasonable time a rapid code. Thus, a prescription for decay times differing slightly from that of Section 3, was adopted. Section 4 also contains an outline of the logical flow of the code. Section 5 contains a listing of the current code together with the output of a sample run.

Section 1
ENERGY LOSS OF FISSION PARTICLES RELEASED AT HIGH ALTITUDES

In this section the different mechanisms contributing to the stopping power of the atmosphere on neutral and ionized fission particles are reviewed. The slowing down of a fission particle is due to its elastic and inelastic collisions with the ambient atoms, and of the latter collisions we consider those giving rise to ionization, electron capture, and electron loss.

For particle velocities below about 5×10^7 cm/sec elastic collisions dominate, the energy loss being governed by the transport cross section through the law

$$\frac{dv}{dt} = -\frac{n(s)}{2}\,\sigma_{tr}\,v^2 \tag{1.1}$$

where $n(s)$ is the number density of the ambient medium. The evaluation of σ_{tr} depends essentially on the interaction potential between the colliding systems. For heavy atoms, the potential can be calculated to an accuracy of 10 percent on the basis of the statistical model for the electrons (Ref. 1). Thus,

$$U(r) = \frac{Z_1 Z_2 e^2}{r}\,\chi(\psi\, r/a) \tag{1.2}$$

where $\chi(x)$ is the screening function in the Thomas-Fermi potential, Z_1 and Z_2 are the atomic numbers of the interacting atoms, r is the internuclear distance,

$$a = \left(9\pi^2/128\right)^{1/3} \hbar^2/me^2 = 4.7 \times 10^{-9}\ \text{cm} \tag{1.3}$$

and

$$\psi = \left(Z_1^{1/2} + Z_2^{1/2}\right)^{2/3} \qquad (1.3)'$$

Using the interaction potential (1.2), Firsov (Ref. 2) obtains an expression for σ_{tr} which he approximates by

$$\sigma_{tr} = 2\pi e^4 M_1 M_2 \left(Z_1 Z_2/E_{cm}(M_1 + M_2)\right)^2 \ln\left(1 + \frac{0.7\, E_{cm}}{30.5\, Z_1 Z_2 \psi}\right) \qquad (1.4)$$

where E_{cm} is the energy in ev in the center of mass system:

$$E_{cm} = \mu E/M_1$$

$$\mu = M_1 M_2/(M_1 + M_2)$$

The resultant stopping power* is

$$W_E = 1.3 \times 10^{-13} \frac{Z_1^2 Z_2^2}{M_2 E/M_1} \ln\left(1 + \frac{0.23\, \mu E}{M_1 Z_1 Z_2 \psi}\right) \mathrm{cm}^2\, \mathrm{ev} \qquad (1.5)$$

To obtain the energy loss due to excitation and ionization, Firsov (Ref. 3) assumes that there is a friction-like force between the orbital electrons of the two atoms while they are passing each other. The resultant conversion of the kinetic energy of relative motion into excitation energy, is due to the transfer of momentum by electrons

*Stopping power is the average energy loss per incident particle per centimeter path in a gas of unit number density.

from one particle to the other in the region where the electronic shells overlap. The energy lost to electron excitation or ionization at a given impact parameter R_o (cm) and velocity V (cm/sec) is

$$\epsilon = \frac{(Z_1 + Z_2)^{5/3} \times 4.3 \times 10^{-8} v}{\left[1 + 3.1(Z_1 + Z_2)^{1/3} 10^7 R_o\right]^5} \tag{1.6}$$

Integrating over all impact parameters to obtain the total stopping power gives

$$W_E = 2\pi \int_0^\infty \epsilon \, R_o \, d R_o = 0.234 \times 10^{-22} (Z_1 + Z_2) v \ \text{cm}^2 \ \text{ev} \tag{1.7}$$

The ionization cross section at a velocity v is given by

$$\sigma = \sigma_o \left[(v/v_o)^{1/5} - 1 \right]^2 \tag{1.8}$$

with

$$\sigma_o = \frac{32.7}{(Z_1 + Z_2)^{2/3}} 10^{-16} \ \text{cm}^2$$

and

$$v_o = \frac{23.3 \ E_i}{(Z_1 + Z_2)^{5/3}} 10^6 \ \text{cm/sec} \tag{1.9}$$

E_i in (ev) being the first ionization potential of the gas atom. The variation of σ with velocity was computed from Eq. (1.8) for $Z_1 = 45$ $Z_2 = 7$:

v (cm/sec)	$\sigma(cm^2 \times 10^{16})$
3×10^6	0.499
5×10^6	0.897
8×10^6	1.421
1×10^7	1.732
3×10^7	4.064
5×10^7	5.748

A comparison of the predictions of Eq. (1.8) with experiment has been carried out by Fedorenko (Ref. 4). The velocity range of the colliding ions varied from 7×10^6 to 9×10^7 cm/sec, and in this region there is agreement between theory and experiment to within a factor of 2. Equation (1.8) is even useful for the lighter atoms, where one finds it to agree to within a factor of 3. A comparison of the theory with the experimental data of Ref. 5 for electron loss cross sections for lithium in helium:

$$Li + He \rightarrow Li^+ + He + e^- + \Delta E \tag{1.10}$$

is made below

v (cm/sec)	$\sigma(cm^2 \times 10^{-16})$ (Firsov)	$\sigma(cm^2 \times 10^{-16})$ (expt)
4×10^7	1.45	0.6
6×10^7	2.52	1.0
8×10^7	3.54	1.3

A comparison of the stopping powers due to elastic collisions with those due to ionizing collisions may be made with the help of Eqs. (1.5) and (1.7). As a typical example, consider a fission ion of atomic mass 100 and atomic number 45. The stopping powers are:

v (cm/sec)	W_E (cm^2 ev) (elastic)	W_E (cm^2 ev) (inelastic)
4×10^7	1.655×10^{-13}	4.69×10^{-14}
1×10^7	1.79×10^{-13}	1.17×10^{-14}

The stopping power due to inelastic scattering falls off rapidly with decreasing energy, being 30 percent of that due to elastic scattering at a velocity of 4×10^7 cm/sec, and only 7 percent at a velocity of 1×10^7 cm/sec.

The stopping power due to electron capture (charge transfer) is more difficult to estimate. For lack of sufficient theoretical or experimental data, it is necessary to rely on empirical rules. From an examination of available experimental data on charge transfer reactions, Hasted (Ref. 6) has observed that the maximum charge-transfer cross section occurs at a velocity v_m of the incident particle given by

$$\frac{a\,\Delta E}{v_m\,h} \approx 1 \qquad (1.11)$$

where ΔE is the energy defect measured in electron volts and "a" has the dimension of length. Hasted claims the best fit is given by $a = 8$ A.

If we express ΔE in ev,

$$v_m = 1.9 \times 10^7 \ \Delta E \ \text{cm/sec} \qquad (1.12)$$

For velocities in the adiabatic region, the charge transfer cross section varies according to

$$\sigma = c \exp(-k\,a\,\Delta E/v) \qquad (1.13)$$

where c and k are constants for any particular reaction. There is, of course, no rigorous theoretical basis for Eq. (1.13). For velocities, greater than that for which σ_{max} occurs, σ falls off with increasing energy as some inverse power of v .

For a typical reaction

$$D^+ + A \rightarrow D + A^+$$

the energy difference will be of the order of 7 ev , so that the velocity at which σ_{max} occurs is approximately 1.3×10^8 cm/sec . It would thus appear that σ_{max} is reached for velocities well outside the range of velocities with which we are concerned.

The stopping power due to capture is

$$\sum_n E_n \sigma_{c,n} + \frac{mE}{M_1} \sum_n \sigma_{c,n}$$

For a velocity of 4×10^7 cm/sec , E = 83.6 kev

and

$$\frac{mE}{M_1} = 0.455 \text{ ev}$$

The value of σ_{max} will be of order of 10^{-16} cm^2 and, at a velocity of 4×10^7 cm/sec, σ should be much less than σ_{max} according to Eq. (1.13). Nevertheless, to obtain a gross overestimate of the effect of charge exchange on the energy loss, let us assume that at $v = 4 \times 10^7$ cm/sec, σ is 10^{-15} cm^2 . Assuming E_n to be approximately 10 ev , we find the stopping power due to charge transfer must be less than 1×10^{-14} cm^2 ev , which is less than that due to inelastic scattering. Thus, energy losses due to charge transfer can be neglected.

Neglecting the energy losses due to inelastic collisions altogether, we expand the logarithm in Eq. (1.4) to obtain

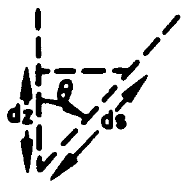
$$\sigma_{tr} \sim \frac{a^2}{x^2} \frac{1}{E} \quad \text{i.e.,} \quad \sigma_{tr} \sim 1/v^2 \tag{1.14}$$

From Eqs. (1.1) and (1.14)

$$\frac{dv}{dt} = - K N \tag{1.15}$$

where K is a function of A and Z , and N is the number density of the atmosphere. Because K is a slowly varying function of its arguments, the error involved in working with $K = 1.8 \times 10^{-3}$ (the value for a typical fragment of $A = 100$ and $Z = 45$) will not be great.

For a particle moving along a straight line path making an angle θ with the vertical



$$ds = ds \cos\theta \tag{1.16}$$

Introducing the density of the atmosphere $\rho(z)$ and

$$N = \frac{A' \rho(z)}{M} \tag{1.17}$$

where A' is Avogadro's number and M the mean molecular weight of the atmospheric atoms, we can write Eq. (1.15)

$$\frac{dv}{dt} = - \frac{K A'}{M \cos\theta} \rho(z)$$

which can be integrated to give

$$v^2 = v_o^2 - \frac{2K\,A'}{M\cos\theta}\int_{z_o}^{z} \rho(z)\,dz \tag{1.19}$$

The precise form of the stopping law used depends upon the assumptions made concerning $\rho(z)$ in evaluating Eq. (1.19). The simplest, and the one adopted here is that the hydrostatic condition

$$dp = -g\,\rho(z)\,dz \tag{1.20}$$

is satisfied. Then, if we neglect the variation of g with altitude, Eq. (1.19) gives

$$v^2 = v_o^2 + \frac{2K\,A'}{M\,g\cos\theta}\Big(p(z) - p(z_o)\Big) \tag{1.21}$$

Section 2
INELASTIC COLLISIONS OF FISSION PARTICLES IN THE ATMOSPHERE

2.1 THE CHARGE STATE OF A BEAM OF PARTICLES

According to the results of the preceding section, the electron capture and loss cross sections do not contribute to the energy losses of particles moving through the atmosphere at velocities equal to or less than about 5×10^7 cm/sec. Nonetheless, these cross sections can be of importance in determining the overall charge state of a collection of particles.

Let n_i and n denote the equilibrium number densities of the debris ions and atoms, respectively. Neglecting the formation of more highly ionized states and negative ions of the debris we have in equilibrium

$$n_i \sigma_c = n\sigma_\ell \tag{2.1}$$

where σ_c and σ_ℓ are the capture and loss cross sections. The charged fraction of the beam is

$$f(i) = \frac{n_i}{n + n_i} = \frac{1}{1 + \sigma_c/\sigma_\ell} \tag{2.2}$$

For a typical electron loss reaction

$$D + A \rightarrow D^+ + e^- + A + \Delta E$$

we can use Eq. (1.8) with E_i of the order of 7 ev. The capture cross section can be estimated with the help of Hasted's (Ref. 7) empirical relation analogous to Eq. (1.13)

$$\sigma_c = c \exp\left\{- a\Delta E/4hv\right\} \tag{2.3}$$

Taking $a\Delta E/4h \simeq 3.3 \times 10^7$ for a typical charge transfer reaction

$$D^+ + A \rightarrow A^+ + D$$

we have the following

V(cm/sec)	$\sigma_\ell (cm^2 \times 10^{-16})$	$\sigma_c (cm^2 \times 10^{-16})$
4×10^7	7.64	.44
1×10^7	2.96	.037

We see that σ_c falls relatively more rapidly with energy than does σ_ℓ, and over the range of velocities of interest $\sigma_\ell > \sigma_c$. Hence, from Eq. (2.2), the fraction of the beam which remains charged is greater than 1/2. There is some experimental confirmation of this result (Ref. 8).

2.2 RANDOM INELASTIC COLLISIONS

In addition to making rough estimates like Eq. (2.2) possible, the inelastic cross sections determine the probability for a particle to undergo a charge-changing collision. To determine the probability distribution for such a collision, we consider a neutral particle moving through a medium of density N along a path s making an angle θ with the vertical. Let σ be the cross-section for ionization of the particle.

The change in the number n of neutral particles along a segment of path ds is given by the well-known attenuation law

$$dn = - n\sigma N ds \tag{2.4}$$

and, using Eqs. (1.16) and (1.17), this can be written

$$\frac{dn}{n} = - \frac{B}{\cos\theta} \rho(z)\, dz \tag{2.5}$$

with

$$B = \frac{\sigma A'}{M} \tag{2.6}$$

Integrating Eq. (2.5) from an arbitrary initial altitude z_o to an arbitrary final altitude z, we have

$$n/n_o = \exp\left\{ - \frac{B}{\cos\theta} P(z, z_o) \right\} \tag{2.7}$$

where

$$P(z, z_o) = \int_{z_o}^{z} \rho(z)\, dz \tag{2.8}$$

Because $\cos\theta$ and $P(z, z_o)$ have the same sign and are negative for particles moving downward, n/n_o is always a decreasing exponential. Observe that the integral Eq. (2.8) also occurs in the stopping law, Eq. (1.19).

Let n_c be the number of particles colliding with the ambient atoms in the interval (z_o, z). The fraction

$$\frac{n_c}{n_o} = 1 - \frac{n}{n_o} \tag{2.9}$$

is the probability for a particle to have an ionizing collision before reaching the altitude z. Differentiating

$$\frac{n_c}{n_o} = 1 - \exp\left\{ - \frac{B}{\cos\theta} P(z, z_o) \right\} \tag{2.10}$$

gives the spatial rate of collision

$$\frac{dn_c}{n_o} = \frac{B}{\cos\theta} \exp\left\{ - \frac{B}{\cos\theta} P(z, z_o) \right\} \rho(z)\, dz \tag{2.11}$$

The density $\rho(z)$ is defined in $0 < z < \infty$. If the integrated density diverges, that is if

$$P(\infty, z_o) = \int_{z_o}^{\infty} \rho(z)\, dz = +\infty \tag{2.12}$$

then Eq. (2.11) is a normalized probability density for collisions and Eq. (2.10) is the corresponding probability distribution. If the integrated density $P(\infty, z_o)$ is finite, the fraction of particles escaping to infinity <u>without</u> undergoing an ionizing collision is finite:

$$\frac{n_\infty}{n_o} = \exp\left\{-\frac{B}{\cos\theta} P(\infty, z_o)\right\} \tag{2.13}$$

the fraction colliding in the atmosphere is $1 - n_\infty/n_o$; and hence, the probability distribution for collisions in the open interval $z_o < z < \infty$ is

$$\frac{1 - n_o}{1 - n_\infty/n_o} \tag{2.14}$$

The standard Monte Carlo procedure is to choose a random number R from the uniform distribution on $(0,1)$ and to solve the equation

$$R = \frac{1 - n/n_o}{1 - n_\infty/n_o} \tag{2.15}$$

for the altitude z at which a collision occurs. A collection of altitudes so determined has a distribution approximating Eq. (2.14). Calculating z directly from Eq. (2.15) can lead to complicated and slow computations because we have to solve the equation

$$\int_{z_o}^{z} \rho(z)\, dz = \frac{-\cos\theta}{B} \ln\left[1 - R\left(1 - \exp\left\{-\frac{B}{\cos\theta}\int_{z_o}^{\infty} \rho(z)\, dz\right\}\right)\right] \tag{2.16}$$

for z.

Alternatively, we can interpret Eq. (2.10) as a normalized distribution on the closed interval $z_o \le z \le +\infty$ with a jump discontinuity at the point $z = +\infty$. The probability associated with $z = +\infty$ is n_∞/n_o. With the discontinuous distribution, the procedure for determining z is to solve

$$R = 1 - n/n_o \tag{2.17}$$

for z if

$$R < 1 - n_\infty/n_o \tag{2.18}$$

holds and to set $z = +\infty$ if Eq. (2.18) fails. Because R and $1 - R$ are simultaneously uniformly distributed on $(0, 1)$, we solve, in practice,

$$P(z, z_o) = -\frac{\cos\theta}{B} \ln R \tag{2.19}$$

for z if

$$R > n_\infty/n_o \tag{2.20}$$

and set $z = \infty$ if Eq. (2.20) fails.

For a finite atmosphere, one in which $\rho(z)$ is defined for $0 \le z \le H$, we use the same procedure, with the fraction n_H/n_o replacing the fraction n_∞/n_o. In practice, we solve Eq. (2.19) for z and say that the particle has reached H without suffering a collision if $z > H$.

2.3 COLLISIONS IN VARIOUS ATMOSPHERES

The explicit calculation of the collision altitude is uniquely determined by the assumption made on the behavior of $\rho(z)$ with z and the resultant evaluation of Eq. (2.8).

2.3.1 Tabulated Atmosphere

Suppose $\rho(z)$ is known as a table of values of ρ vs. z in an interval $0 < c < z < H$. By a suitable interpolation rule we can determine $\rho(z_o)$ for any z_o in the interval. We can also construct values of $P(c,z)$ and $P(H,z)$, and by interpolation, those of $P(c,z_o)$ and $P(H,z_o)$. Then the probability of escaping over the upper boundary H without a collision between the starting point z_o and H, is

$$n_H/n_o = \exp\left\{-\frac{B}{\cos\theta}\, P(H,z_o)\right\} \tag{2.21}$$

and the probability distribution for collisions in the interval (H,z_o) is

$$\frac{1 - n/n_o}{1 - n_H/n_o} \tag{2.22}$$

with n/n_o given by Eq. (2.7). Equating (2.22) to a random number R and solving for z, yields a collision point distributed according to Eq. (2.14), i.e.,

$$\int_{z_o}^{z} \rho(z)\,dz = -\frac{\cos\theta}{B}\ln\left[1 - R(1 - n_H/n_o)\right] \tag{2.23}$$

Interpolation among the tabulated values of the integrated density gives z.

2.3.2 Constant Atmosphere

Let $\rho(z) = \rho$ a constant in the interval $0 < c < z < H$. In this case

$$P(z,z_o) = \rho(z - z_o) \tag{2.24}$$

and the probability of escaping over the upper boundary is

$$n_H/n_o = \exp\left\{-\frac{B}{\cos\theta}\,\rho(H - z_o)\right\} \tag{2.25}$$

Because the integrated density diverges at infinity, we can replace a finite atmosphere with constant density by an infinite atmosphere with the same constant density. In such an atmosphere, the probability distribution for collisions is simply

$$1 - n/n_o \tag{2.26}$$

Choosing a random number R and solving for z, we get

$$z = z_o + \frac{\cos\theta}{\rho B} \ln R \tag{2.27}$$

If this value of z fails to satisfy

$$c < z < H$$

it is rejected as a possible collision point. Note that the probability of rejecting such a z is

$$\int_H^\infty \frac{B}{\cos\theta} \exp\left\{-\frac{B\rho}{\cos\theta}(z - z_o)\right\} \rho\, dz = \exp\left\{-\frac{B\rho}{\cos\theta}(H - z_o)\right\} \tag{2.28}$$

which is precisely the probability of escape over the upper boundary H .

2.3.3 Linear Atmosphere

Suppose the density decreases linearly according to

$$\rho(z) = b - az \,, \quad a > 0 \tag{2.29}$$

in $0 < c < z < H$. Because $\rho(z) \geq 0$, we have

$$b/a > H \tag{2.30}$$

The integrated density is

$$P(z,z_o) = (z - z_o)\left[b - \frac{a}{2}(z + z_o)\right] \tag{2.31}$$

a quadratic polynomial with zeros at z_o and

$$z = \frac{2b}{a} - z_o \tag{2.32}$$

The fact that $z_o < H$, and Eq. (2.30) imply

$$\frac{2b}{a} - z_o > H \tag{2.33}$$

Hence, the second zero occurs above H. The collision point is determined in the usual manner.

2.3.4 Exponentially Decreasing Atmosphere

Suppose the density is given by

$$\rho(z) = a \exp\left\{- b(z - c)\right\} \tag{2.34}$$

with $a, b > 0$ in $0 < c < z < H$. The integrated density becomes

$$P(z,z_o) = \frac{a}{b}\left[\exp\left\{- b(z_o - c)\right\} - \exp\left\{- b(z - c)\right\}\right] = q(z_o) - q(z) \tag{2.35}$$

with

$$q(z) = \frac{a}{b} \exp\left\{- b(z - c)\right\}$$

The probability of escape over the upper boundary H is now

$$n_H/n_o = \exp\left\{- \frac{B}{\cos\theta}\left[q(z_o) - q(H)\right]\right\} \tag{2.36}$$

and solving the equation

$$R = \frac{1 - n/n_o}{1 - n_H/n_o}$$

for z gives

$$z = c - \frac{1}{b} \ln\left[\frac{b}{a}(q(z_o) - P)\right] \qquad (2.37)$$

where

$$P = -\frac{\cos\theta}{B} \ln\left[1 - R(1 - n_H/n_o)\right]$$

2.3.5 Hydrostatic Atmosphere

The previous assumptions concerning $\rho(z)$ are either manifestly unsuitable or lead to computations which are too slow for Monte Carlo calculations. The hydrostatic relation, Eq. (1.20)

$$dp(z) = -g\rho(z)\,dz$$

where $p(z)$ is the pressure at altitude z and g is the acceleration due to gravity can be integrated to give

$$P(z, z_o) = \frac{p_o - p}{g} \qquad (2.38)$$

where g is assumed constant over the altitudes of interest. The fraction surviving Eq. (2.7) becomes

$$n/n_o = \exp\left\{-\frac{B}{\cos\theta}\,\frac{p_o - p}{g}\right\} \qquad (2.39)$$

Using the alternative procedure, Eqs. (2.19) and (2.20), and solving for the collision pressure rather than the collision altitude gives

$$p = p_o + g\frac{\cos\theta}{B} \ln R \tag{2.40}$$

as a provisional value of p to be accepted only if

$$p > p_H \tag{2.41}$$

Interpolation in the table of p vs z (Section 5) determines the collision altitude. The altitudes so determined have

$$\frac{1 - n/n_o}{1 - n_H/n_o} \tag{2.42}$$

as a normalized probability distribution.

The assumptions usually made in integrating the hydrostatic relation are (i)

$$\rho(z) = \frac{M}{kT}\, p(z) \tag{2.43}$$

where k is Boltzmann's constant, M the molecular weight, and T the temperature; (ii) the variation of g, M, and T can be neglected. In the presence of these assumptions, we easily find that

$$\rho(z) = \rho(z_o) \exp\left\{- \frac{Mg}{kT} (z - z_o)\right\} \tag{2.44}$$

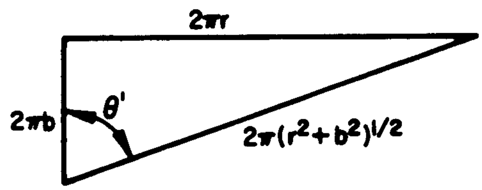
an exponentially decreasing atmosphere, [cf Eq. (2.34)].

2.4 COLLISIONS ALONG A SPIRAL PATH

Let the z' axis of a new rectangular coordinate system have a polar angle θ_o with respect to the z axis. Suppose a particle travels along a spiral path, say the helix

$$\begin{aligned} x' &= r \cos u \\ y' &= r \sin u \\ z' &= bu \end{aligned} \tag{2.45}$$

where r is the radius of gyration and $2\pi b$ is the distance the particle advances along the z' axis in one revolution. If we unroll the path for a single revolution, we get the following sketch



Here θ' is the pitch angle of the spiral. We see that the parameter u is connected with the arc length s by the relation

$$u = s/\left(r^2 + b^2\right)^{1/2} \tag{2.46}$$

and that

$$\cos \theta' = b/\left(r^2 + b^2\right)^{1/2} \tag{2.47}$$

holds. Thus, we have

$$z' = bs/\left(r^2 + b^2\right)^{1/2} = s \cos \theta' \tag{2.48}$$

or

$$dz' = \cos \theta' \, ds \tag{2.49}$$

Moreover, because the polar angle of the z' axis is θ_o we have, with respect to the z axis,

$$dz = \cos\theta_o \, dz' \tag{2.50}$$

Combining Eqs. (2.49) and (2.50) gives

$$dz = \cos\theta_o \cos\theta' \, ds \tag{2.51}$$

Comparing Eq. (2.51) with Eq. (1.16) shows that the analogue for Eq. (2.5) in the case of a spiral path is

$$\frac{dn}{n} = - \frac{B}{\cos\theta_o \cos\theta'} \rho(z) \, dz \tag{2.52}$$

Thus, we may simply replace $\cos\theta$ by the product

$$\cos\theta_o \cos\theta' \tag{2.53}$$

in the formulas for straight line paths to obtain the formulas appropriate for spiral paths.

We also make the same replacement when applying Eq. (1.21) to describe the slowing down along a spiral path.

Section 3
BETA DECAY OF FISSION FRAGMENTS

A correct statistical treatment of beta decay would require a knowledge of all decay chains and the lifetimes and branching ratios of all the elements in these chains. Because such detailed data are not known experimentally, we are forced to make model assumptions. In this we are guided by a few well-established experimental facts about beta decay. According to Way and Wigner (Ref. 9) the combined beta activity of all fission products and succeeding generations stays roughly constant for 1 sec and then drops as $t^{-1.2}$. The length of the decay chains varies, but on the average about three betas are emitted per fission product. We define the beta activity $A(t)$ to be the average number of betas per fission product per unit time so that

$$\int_0^\infty A(t)dt = 3 \tag{3.1}$$

With this normalization, the Way-Wigner law takes the form

$$A(t) = \begin{cases} \frac{1}{2}, & t \leq 1 \text{ sec} \\ \frac{1}{2}t^{-1.2}, & t > 1 \text{ sec} \end{cases} \tag{3.2}$$

In our model we assume that all fission products produce exactly three electrons. For the Monte Carlo method, we need probability distributions for each of the three decays.

Let us introduce the fractions of fission products n_0, n_1, n_2, and n_3, which are present after 0, 1, 2, and 3 decays have taken place. At $t = 0$, no decays have occurred, so that the initial conditions are

$$n_0 = 1 \text{ and } n_1 = n_2 = n_3 = 0 \tag{3.3}$$

Let us introduce three positive functions, $f_1(t)$, $f_2(t)$, and $f_3(t)$, describing the rate of production of betas by first, second, and third decays. The fractions n_i obey differential equations of the form

$$\frac{dn_0}{dt} = -f_1(t)n_0$$

$$\frac{dn_1}{dt} = f_1(t)n_0 - f_2(t)n_1 \tag{3.4}$$

$$\frac{dn_2}{dt} = f_2(t)n_1 - f_3(t)n_2$$

and the normalization condition for all times

$$n_3 = 1 - n_0 - n_1 - n_2 \tag{3.5}$$

Because there are not enough experimental data to determine the three rate functions, we can make the assumption that they have a common value, say $f(t)$.

Let us introduce the scaled time

$$s = \int_0^t f(u)du \tag{3.6}$$

The system [Eq. (3.4)] simplifies to

$$\left.\begin{aligned} \frac{dn_0}{ds} &= -n_0 \\ \frac{dn_1}{ds} &= n_0 - n_1 \\ \frac{dn_2}{ds} &= n_1 - n_2 \end{aligned}\right\} \tag{3.7}$$

which has the solution

$$\left.\begin{aligned} n_0 &= e^{-s} \\ n_1 &= se^{-s} \\ n_2 &= \frac{s^2}{2} e^{-s} \end{aligned}\right\} \qquad (3.8)$$

satisfying the initial conditions required by Eq. (3.3).

We note that

$$\int_0^{\infty} n_i(s)\,ds = 1 , \qquad i = 0, 1, 2, \qquad (3.9)$$

holds for each of Eqs. (3.8). In terms of the scaled time s, the analogue of the rate function f is unity. Consequently, we can identify n_i, $i = 0, 1, 2$ as the probability density of the $(i + 1)$st decay time in scaled time. The corresponding densities in real time are

$$f(t) n_i \int_0^t f(u)\,du \quad , \qquad i = 0, 1, 2 \qquad (3.10)$$

Because first, second, and third decays for an individual particle occur sequentially, we need a procedure for choosing three random vairables t_1, t_2, and t_3 satisfying

$$0 < t_1 < t_2 < t_3 \qquad (3.11)$$

such that t_i is a sample from the distribution of i^{th} decay times Eq. 3.10 . Out strategy is to make use of the one--one, monotone, and continuous relation between s and t, Eq. (3.6), to solve the analogous problem in scaled time, and then to transform the random variable s_i into random variables t_i.

If u_i, $i = 1, 2, 3$, are independent random variables each with e^{-u} as probability density, then the random variable $s_2 = u_1 + u_2$ has $n_1(s) = se^{-s}$ as probability density, and the random vairable $s_3 = u_1 + u_2 + u_3 = s_2 + u_3$ has $n_2(s) = \left(s^2/2\right)e^{-s}$ as probability density. Consequently, we solve the problem in scaled time by choosing three random numbers, R_i, $i = 1, 2, 3$, from the uniform distribution on $(0, 1)$, by setting $u_i = -\ln R_i$, and by taking

$$s_1 = u_1 = -\ln R_1$$

$$s_2 = u_1 + u_2 = s_1 - \ln R_2 \tag{3.12}$$

and $$s_3 = u_1 + u_2 + u_3 = s_2 - \ln R_3$$

Requiring the total activity resulting from all three generations to reproduce the Way-Wigner law yields

$$A(t) = f(t)\left[n_o + n_1 + n_2\right] = \frac{ds}{dt}\left[1 + s + \frac{s^2}{2}\right]e^{-s} \tag{3.13}$$

Integrating Eq. (3.13) with respect to t gives

$$\int_o^t A(t)\,dt = \int_o^s \left[1 + s + \frac{s^2}{2}\right]e^{-s}\,ds \tag{3.14}$$

Under the hypothesis that the Way-Wigner law holds, the relation Eq. (3.14) between s and t is equivalent to Eq. (3.6) but does not involve a knowledge of $f(t)$. Carrying out the integrations in Eq. (3.14) and solving for t gives

$$
t = \begin{cases} 2s & 0 \le s \le 1/2 \\ \left[\dfrac{5}{6 - 2S}\right]^5 , & 1/2 < S \end{cases} \tag{3.15}
$$

with

$$
S = 3 - \left[3 + 2s + \frac{s^2}{2}\right] e^{-s}
$$

Combining Eqs. (3.12) and (3.15) gives decay times t_i , $i = 1, 2, 3,$ satisfying (3.11).

The uniform rate function $f(t)$ is nearly constant for $0 \le t \le 1$ and decreases for $t > 1$. In fact, manipulating Eq. (3.14) yields

$$
f(t) = \begin{cases} \dfrac{1}{2} + \left[\dfrac{e^s - \left(1 + s + \frac{s^2}{2}\right)}{2 + 2s + s^2}\right], & t \le 1 \text{ sec}, \\ \left[1 + \dfrac{4 + 2s}{2 + 2s + s^2}\right]\dfrac{1}{5t} , & t > 1 \text{ sec} \end{cases} \tag{3.16}
$$

with the value of s for which $t = 1$ being approximately 0.51 .

A simpler, alternative way to obtain three decay times for a particle is to choose three samples, t , from the distribution with the Way-Wigner law as probability density, to order the samples so that Eq. (3.11) holds, and then to declare the resulting t_i to be the i^{th} decay time. This procedure amounts to having different rate functions in the differential [Eq. (3.4)] .

Interpreted as a probability density, the Way-Wigner law takes the form

$$w(t) = \begin{cases} 1/6 & , \ t \le 1 \text{ sec} \\ t^{-1.2}/6 & , \ t > 1 \text{ sec} \end{cases} \tag{3.17}$$

and the corresponding probability distribution is

$$W(t) = \begin{cases} t/6 & , \ t \le 1 \text{ sec} \\ 1 - 5t^{-0.2}/6 & , \ t > 1 \text{ sec} \end{cases} \tag{3.18}$$

To obtain a sample from this distribution, we choose a random number, R, from the uniform distribution on $(0, 1)$ and set

$$t = W^{-1}(R) \tag{3.19}$$

Because $R = W(t)$ is a monotone, increasing, and continuous function, ordering the three random numbers, R, to satisfy

$$0 < R_1 < R_2 < R_3 \tag{3.20}$$

is equivalent to ordering the three corresponding samples, t, to satisfy Eq. (3.11). The probability densities for the minimum, middle, and maximum of three independent random numbers are

$$p_1(R) = 3(1 - R)^2$$

$$p_2(R) = 6R(1 - R)$$

$$p_3(R) = 3R^2 \tag{3.21}$$

We regard $R = W(t)$ as a mapping from real time, t, to another scale time, R. In the alternative procedure, the $p_i(R)$, $i = 1, 2, 3$, are the densities of i^{th} decays in scale time, R. The corresponding rates at which the fractions n_0, n_1, and n_2 decrease are

$$\begin{aligned} \frac{dn_0}{dR} &= - p_1(R) \\ \frac{dn_1}{dR} &= p_1(R) - p_2(R) \\ \frac{dn_2}{dR} &= p_2(R) - p_3(R) \end{aligned} \tag{3.22}$$

Integrating Eq. (3.22) with the initial conditions (3.3) gives

$$\begin{aligned} n_0 &= (1 - R)^3 \\ n_1 &= 3R(1 - R)^2 \\ n_2 &= 3R^2(1 - R) \end{aligned} \tag{3.23}$$

By introducing three different rate functions

$$\begin{aligned} g_1(R) &= \frac{3}{1 - R} \\ g_2(R) &= \frac{2}{1 - R} \\ g_3(R) &= \frac{1}{1 - R} \end{aligned} \tag{3.24}$$

we can rewrite Eq. (3.22) as

$$\frac{dn_0}{dR} = - g_1(R)\, n_0$$

$$\frac{dn_1}{dR} = g_1(R)\, n_0 - g_2(R)\, n_1$$

$$\frac{dn_2}{dR} = g_2(R)\, n_1 - g_3(R)\, n_2 \tag{3.25}$$

which is an analog of the system [Eq. (3.7)] in the first model.

By using Eq. (3.19) to transform Eq. (3.25) from scale time, R, to real time, t, we get a system of differential equations of the form Eq. (3.4) in which the rate functions are

$$f_1(t) = \frac{3}{6 - t}$$

$$f_2(t) = \frac{2}{6 - t}$$

$$f_3(t) = \frac{1}{6 - t} \tag{3.26}$$

for $0 \leq t \leq 1$ and

$$f_1(t) = 3/5t$$

$$f_2(t) = 2/5t$$

$$f_3(t) = 1/5t \tag{3.27}$$

for $t > 1$. These rate functions increase for $0 \leq t \leq 1$ and decrease as $1/t$ for $t > 1$.

We adopt the procedure of Eqs. (3.18, 19, and 20)as the model for beta decays primarily because it leads to simpler computations. Moreover, the rate of successive decays decreases. For a typical fission fragment this behavior of decay rates seems intuitively plausible because of the excess of the neutron-to-proton ratio of the particle, which gives rise to its instability, decreases with successive decays.

The graphs of injection density in Ref. 10 are based upon still another prescription for the decay times. In those computations we used the relation

$$t = \left[\frac{5 \exp s}{6 + 4s + s^2}\right]^5 + \left(\frac{5}{6}\right)\left[\frac{s}{0.4 + 10s^3} - \frac{s}{1 + 10s^2}\right] \tag{3.28}$$

with

$$s = s_i = -\ln R_i , \qquad i = 1, 2, 3 \tag{3.29}$$

to compute the decay times. Ordering the random numbers R_i , chosen from the uniform distribution on $(0, 1)$, to satisfy

$$0 < R_3 < R_2 < R_1 < 1 \tag{3.30}$$

insures that the t_i increase with i . Equation (3.28) is an analytic approximation for the two relations in Eq. (3.15).

For a triple of random numbers ordered as in Eq. (3.30), $s_3 = -\ln R_3$ is the largest value produced by Eq. (3.29); for the same triple, $s_3 = -(\ln R_1 + \ln R_2 + \ln R_3)$ is the largest value produced by Eq. (3.12). Thus it is clear that the third decay times produced by the prescription of Eqs. (3.28), (3.29), and (3.30) must be smaller than those produced by the prescription of Eqs. (3.12) and (3.15).

What is true for third decays holds in general: the total distribution of decay times, without regard for decay number, produced by the present prescription has many more early times and for fewer late times than do the distributions produced by the two previous prescriptions, which approximate the Way-Wigner law.

Figures 1 and 2 show, for two sets of parameters, the variation of injection density with distance for the three prescriptions for the decay times; Figures 3 and 4 show the variation of injection density with altitude for the same data. With 2.5×10^4 histories, the results obtained from Eqs. (3. 12) and (3. 15) are almost identical with those obtained from Eqs. (3. 18), (3. 19), and (3. 20). The effect of the prependerance of early decay times in the third prescription, Eqs. (3. 28), (3. 29), and (3. 30), is to suppress the tails and to accentuate the central peak in the distributions of injection density.

We observe that the parameter change (increasing the velocity) broadens the central peak for all prescriptions and that the broadening is greatest for the third prescription.

The discussion of the relative influence of the various parameters in Section 3 of Ref. 10 is based upon the comparative shapes of the injection density curves and is facilitated by using the third prescription to magnify these effects. On the other hand effects which occur at late times are minimized by the third prescription. Our computations suggest that the assumption of a uniform decay rate, the first prescription, also tends to obscure effects occurring at late times. Consequently, for statistics concerning effects at late times, we recommend the second prescription for decay times, Eqs. (3. 18), (3. 19), and (3. 20). This second prescription is in the current version of the code.

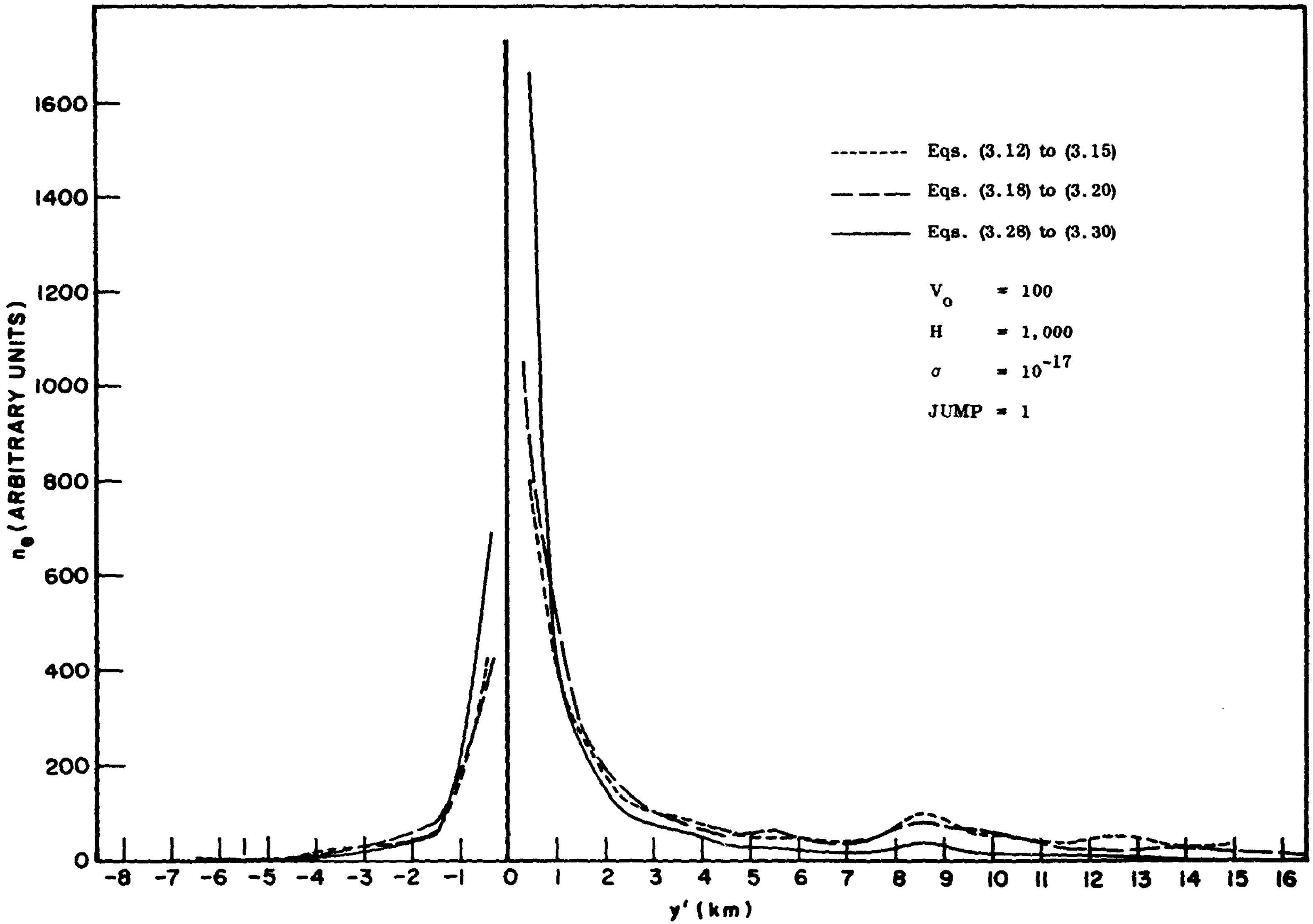


Fig. 1 Variation of Injection Density With Distance for Different Prescriptions of Decay Times

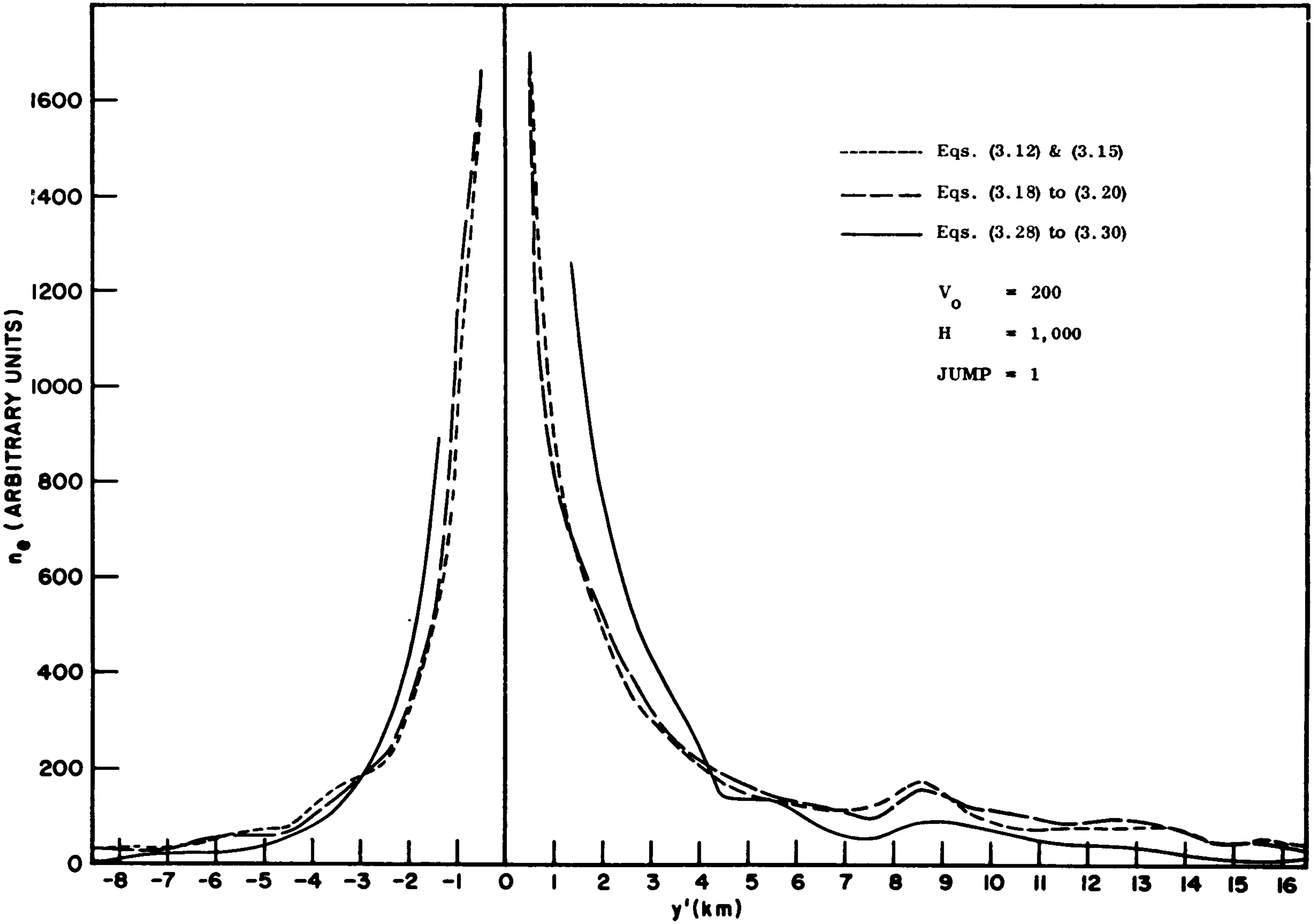


Fig. 2 Variation of Injection Density With Distance for Different Prescriptions of Decay Times

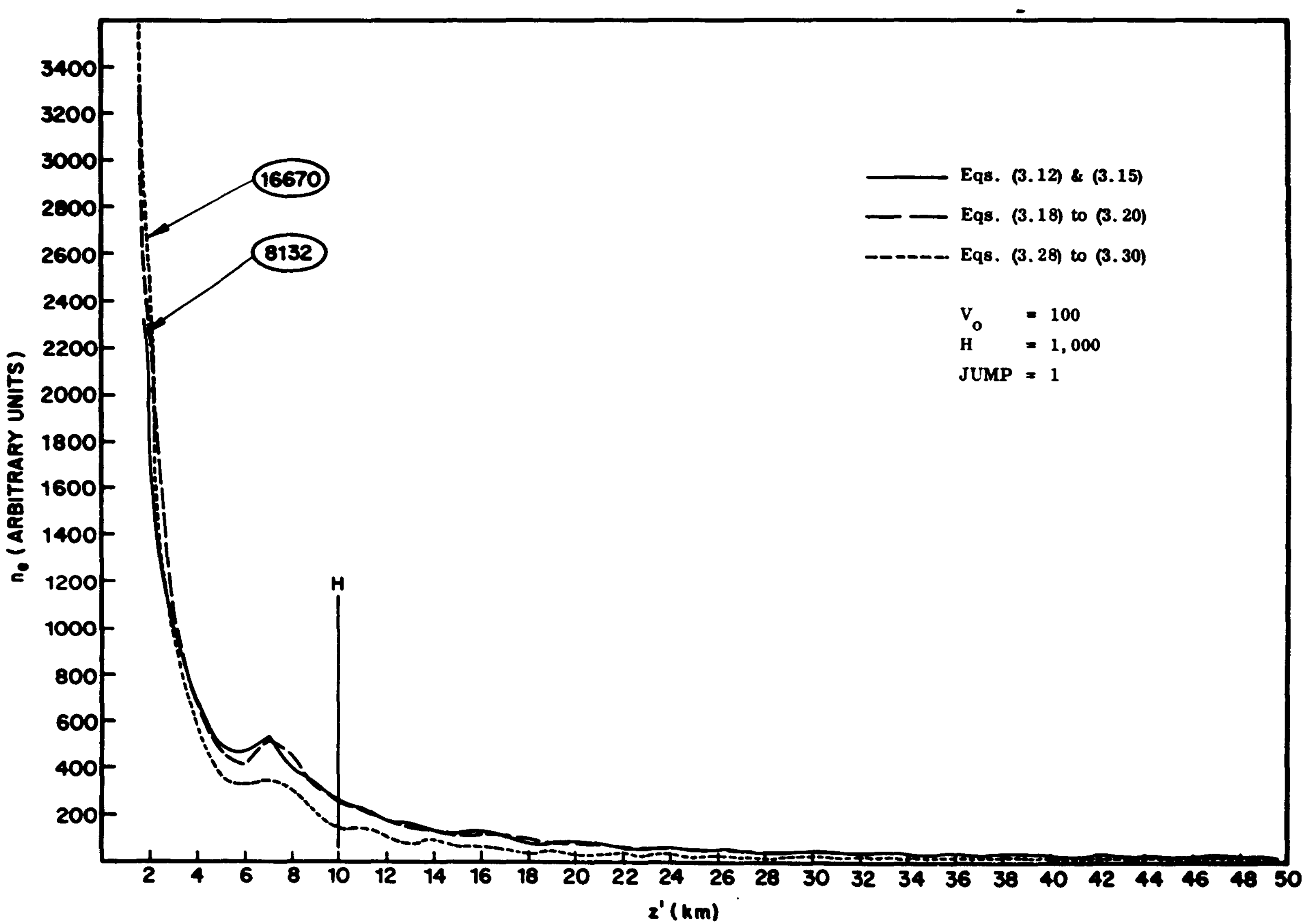


Fig. 3 Variation of Injection Density With Altitude for Different Prescriptions of Decay Times

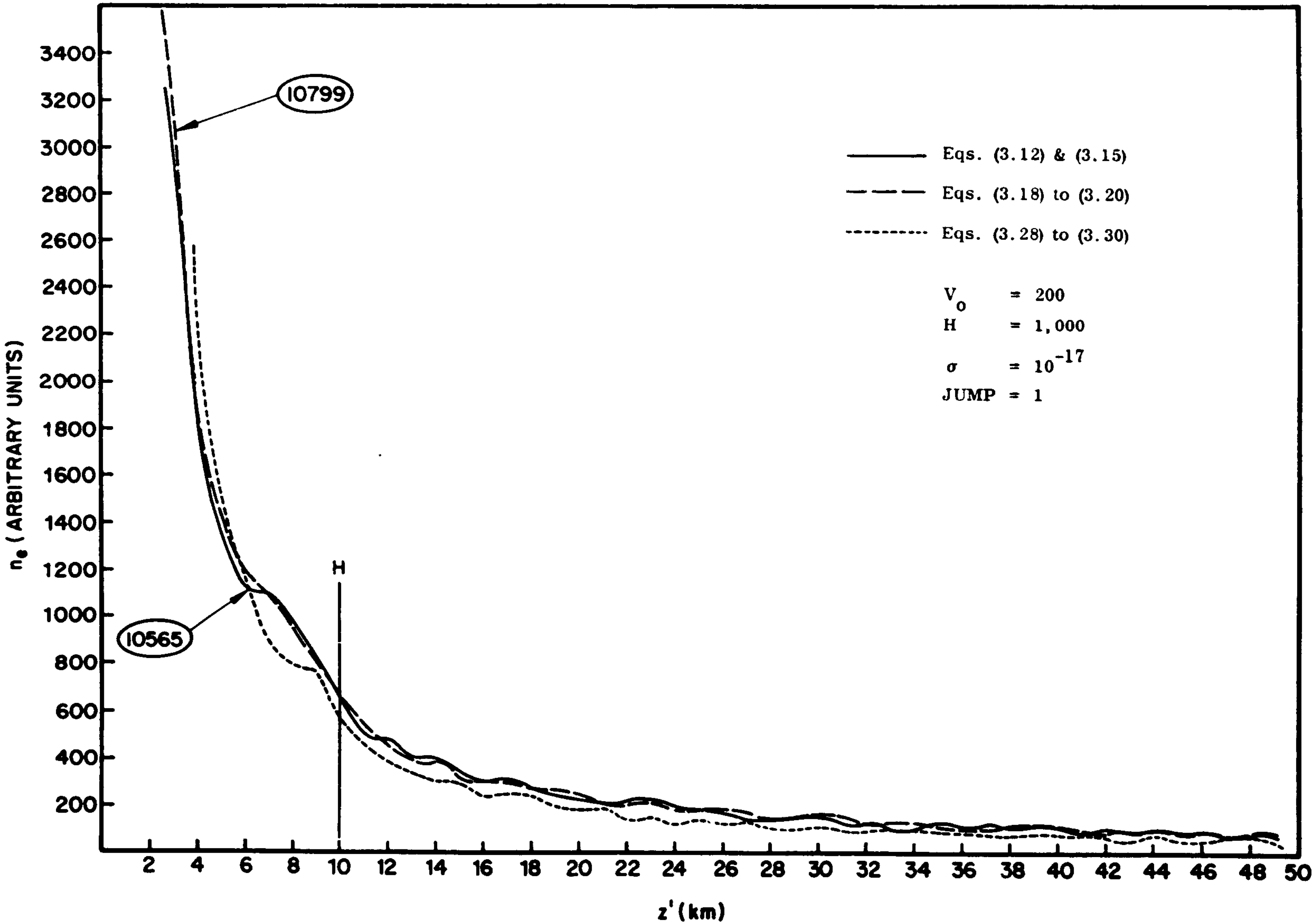


Fig. 4 Variation of Injection Density With Altitude for Different Prescriptions of Decay Times

Section 4

IBM 7090 CODE FOR THE MONTE CARLO MODEL

The fraction of the beam which is ionized is a free parameter and constitutes one of the input constants of the problem. A random number chosen between 0 and 1, compared with this fraction, decides whether the particle considered is neutral or ionized. From then on, a special counter (I) keeps track of the charge on the particle.

The three decay times are computed from three random numbers chosen independently from a uniform distribution on (0, 1) and then ordered to satisfy

$$0 < R_1 < R_2 < R_3 < 1 \tag{4.1}$$

The decay times are obtained by the prescription, derived from Eqs. (3.18) and (3.17),

$$t_i = \begin{cases} 6R_i & , \text{ if } R_i \leq 1/6 \\ \left[\dfrac{5}{6(1 - R_i)}\right]^5 & , \text{ if } R_i > 1/6 \end{cases} \tag{4.2}$$

A particle initially neutral is taken as traveling in a straight line in a direction chosen from a uniform distribution on a hemisphere. Figure 1 shows the coordinates of the particle with θ and φ taken as uniformly distributed between 0 and π ; and 0 and 2π , respectively. On ionization by collision or beta decay, the particle is constrained to spiral about the magnetic field line passing through the point at which ionization occurs, thus neglecting the lateral displacement equal to the radius of gyration. Because the latter is an order of magnitude lower than the mean free path, the resultant error in the coordinates of the fragment will, on the average, be negligible. To follow the motion of the ionized particles, we go to a coordinate system in which the z'-axis is parallel to the magnetic field through the origin. In this (magnetic coordinate) system, the polar angle of the particle position θ' is also its pitch angle.* For the geometry of Fig. 5 we get

$$\cos \theta' = \cos \theta \cos \theta_0 - \sin \theta \sin \theta_0 \sin \varphi \tag{4.3}$$

$$\cos \varphi' = (\sin \theta / \sin \theta') \cos \varphi \tag{4.4}$$

Also, from the figure, we see that if the particle moves a distance ds along the spiral it will rise through a verticle distance given by

$$dz = ds \cos \theta' \cos \theta_0 \tag{4.5}$$

This equation, which is the same as Eq. (2.51), is used to obtain the effective path length, because, in spiralling, the ionized particle has a greater probability of collision than a neutral particle moving through the same verticle distance. The ionized particle remains on the spiral until it is slowed down to rest, neutralized by collision, or ceases to be of interest because it has decayed three times. If it remains on the spiral, it is followed to the conjugate point. The coordinates of every decay point are noted, but the motion of the ionized particle remains unaffected.

*For initially ionized particles, θ' is chosen from a uniform distribution on a hemisphere with the polar axis parallel to the magnetic field,and the calculation proceeds as above.

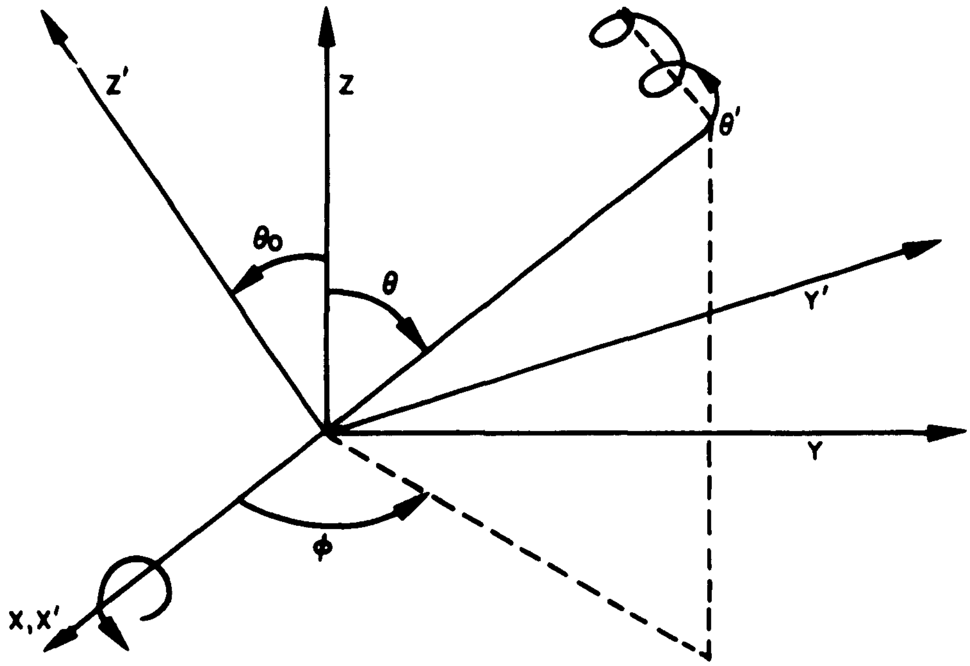


Fig. 5 Coordinate Systems Used

Throughout its passage through the atmosphere the particle's velocity is reduced according to Eq. (1.21) until it escapes with the velocity it has at the altitude H. Let v_{n-1}, v_n and p_{n-1}, p_n be the values of the velocity and pressure at any two consecutive points z_{n-1}, z_n along the path. Generalizing Eq. (1.21), we have

$$v_n^2 = v_{n-1}^2 + \frac{2kA'}{Mg\cos\theta}(p_{n-1} - p_n) \tag{4.5}$$

Similarly, generalizing Eq. (2.40) yields

$$p_n = p_{n-1} + \frac{Mg\cos\theta}{\sigma_i A'} \ln R \tag{4.6}$$

where use has been made of Eq. (2.6). Combining Eqs. (4.5) and (4.6) gives the alternative form for the slowing down law:

$$v_n^2 = v_{n-1}^2 + \frac{2k}{\sigma_i} \ln R \tag{4.7}$$

In the code we use the notation,

$$\frac{Mg\cos\theta}{A'\sigma_i} = A_i\,, \quad \frac{2kA'}{Mg\cos\theta} = B\,, \quad \text{and} \quad \frac{2k}{\sigma_i} = C_i \tag{4.8}$$

We note the appearance of the inelastic collision cross section σ_i in Eq. (4.7), although the corresponding collisions are ignored as sources of energy loss. The explanation is immediate if we remember that the slowing down of a particle must be proportional to its path length. The effect of different types of inelastic collision cross sections can be studied by varying the constants A_i and C_i. The effect of various elastic collision cross sections could be investigated by varying k in the constants B and C_i.

A collision with an atmospheric atom reduces the charge on the particle by unity, the probability for electron capture being definitely greater than that for loss because of the relatively high values of the ionization potentials beyond the first. The motion of the neutralized particle is followed by reverting to the unprimed coordinate system by means of the transformation inverse to Eqs. (4.3) and (4.4), viz.,

$$\cos\theta = \cos\theta' \cos\theta_o + \sin\theta' \sin\theta_o \sin\theta' , \qquad (4.9)$$

$$\cos\theta = (\sin\theta'/\sin\theta) \cos\theta' \qquad (4.10)$$

where θ' is the azimuthal angle in the magnetic coordinate system and is chosen from a uniform distribution between 0 and π.

Decays and inelastic collisions are independent,and therefore the time intervals to decays and collisions are used to determine which event occurs first. By means of a random number, the pressure at the point of collision is determined; the pressure is a known function of the altitude which can be determined from the tables reproduced for convenience, in Section 6. Hence, the distance travelled and the time to cover this distance are found,and this time is compared with the time to first decay. If the time to decay is shorter, the time to collision is reduced by this amount and the residue compared with the time to second decay. If the time to collision is shorter, the time to first decay is reduced, and a new collision time is calculated and compared with it.

The need for two slowing down equations arises as follows. Consider a neutral particle at a starting point z_{n-1} and suppose it is to undergo an ionizing collision at a point z_n. We determine p_n and hence v_n by either Eq. (4.5) or Eq. (4.7). Suppose, however, we find that a decay will occur enroute to z_n, at z_n'. There is no direct method of determining this point and so we proceed in the following way: we calculate the average velocity from z_{n-1} to z_n and, with the time to decay, find the path length to z_n and hence z_n' and p_n'. To proceed with the history we require v_n'. It would clearly be very inaccurate to use v_n or the above average because the point z_n' can be far below z_n and very close to z_{n-1}. Equation (4.7) is now useless and we have to use Eq. (4.5).

At each calculation of the collision time, the new pressure is compared with that at the altitude H to determine whether the particle escapes from the atmosphere. Neutral particles which would escape are checked for possible decays before they could do so, because in case of decay, their path in the atmosphere is lengthened, they are slowed down more, and might even be stopped. Neutral particles which do escape are checked for decays up to the altitude of 20,000 km above which they are considered to be lost.

The calculation thus proceeds step by step until the history of a particle is terminated by one of the following:

- three decays
- loss at the conjugate point
- loss in atmosphere by slowing down to rest
- loss in atmosphere by scattering in the backward direction ($\cos\theta$ or $\cos\theta'$ negative in Eq. (4.4) or (4.9)
- loss by escaping from the atmosphere and traveling out of region of interest (neutral or neutralized particles only)

The code is set out in eight blocks of instructions, each designed to cover a particular aspect of a particle's history according to the following scheme:

Instruction Set No.	Function
100	Initialization
200	Collision Routine for Neutral Particles (CN)
300	Collision Routine for Ionized Particles (CI)
400	Decay Routine for Neutral Particles (DN)
500	Decay Routine for Ionized Particels (DI)
600	Escape Routine for Neutral Particles (EN)
800	Escape Routine for Ionized Particles (EI)
6600	Escape Routine for Neutral Particles with zero ionization cross section (EN°)

The details of all the steps involved are shown in the flow charts of Figs. 6 through 16.

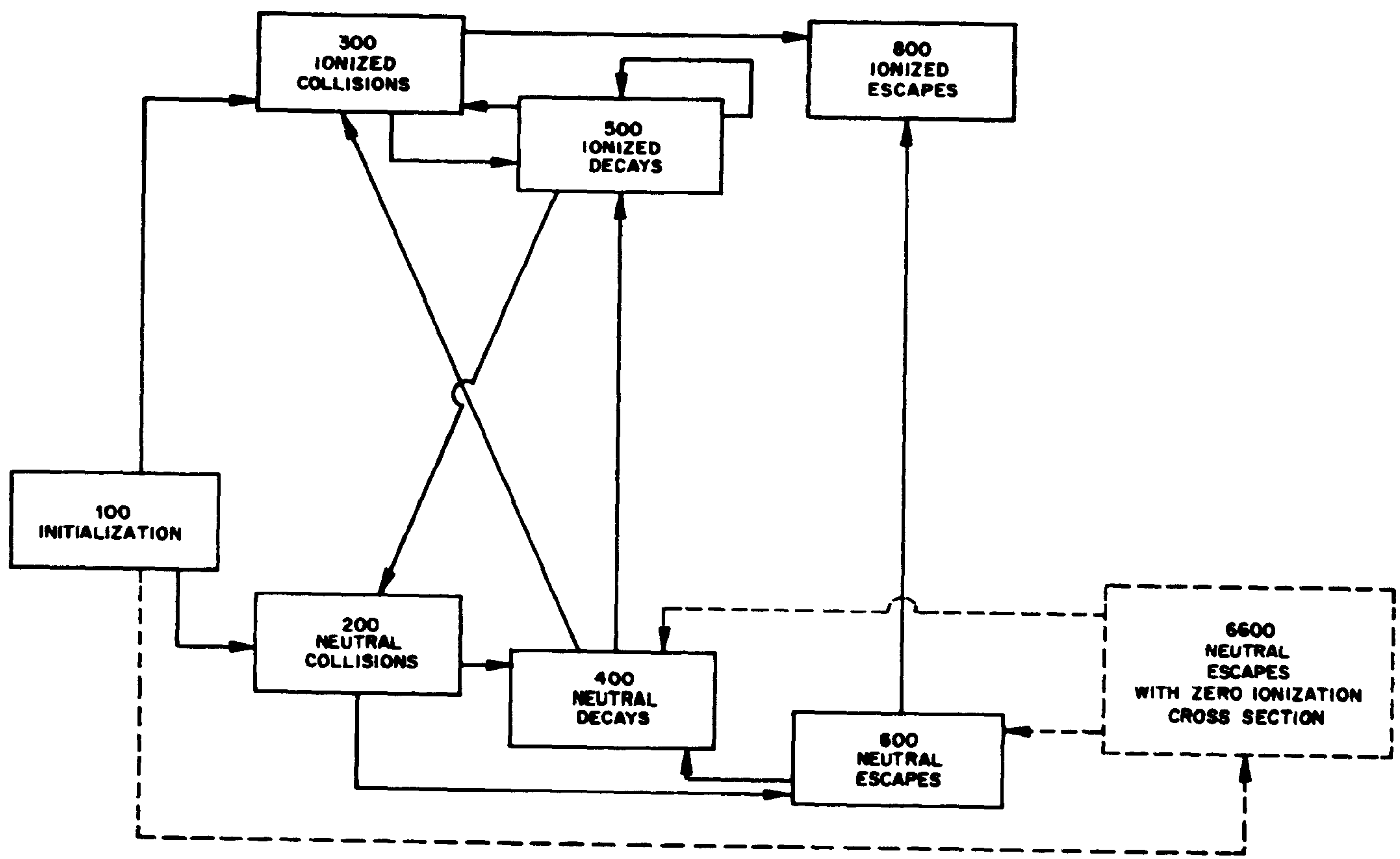


Fig. 6 Some Possible History Paths in Terms of the Coding Blocks. (Dashed lines refer to initial stages of calculations with zero ionization cross sections. Subroutines, exits, and history terminations not shown for clarity.)

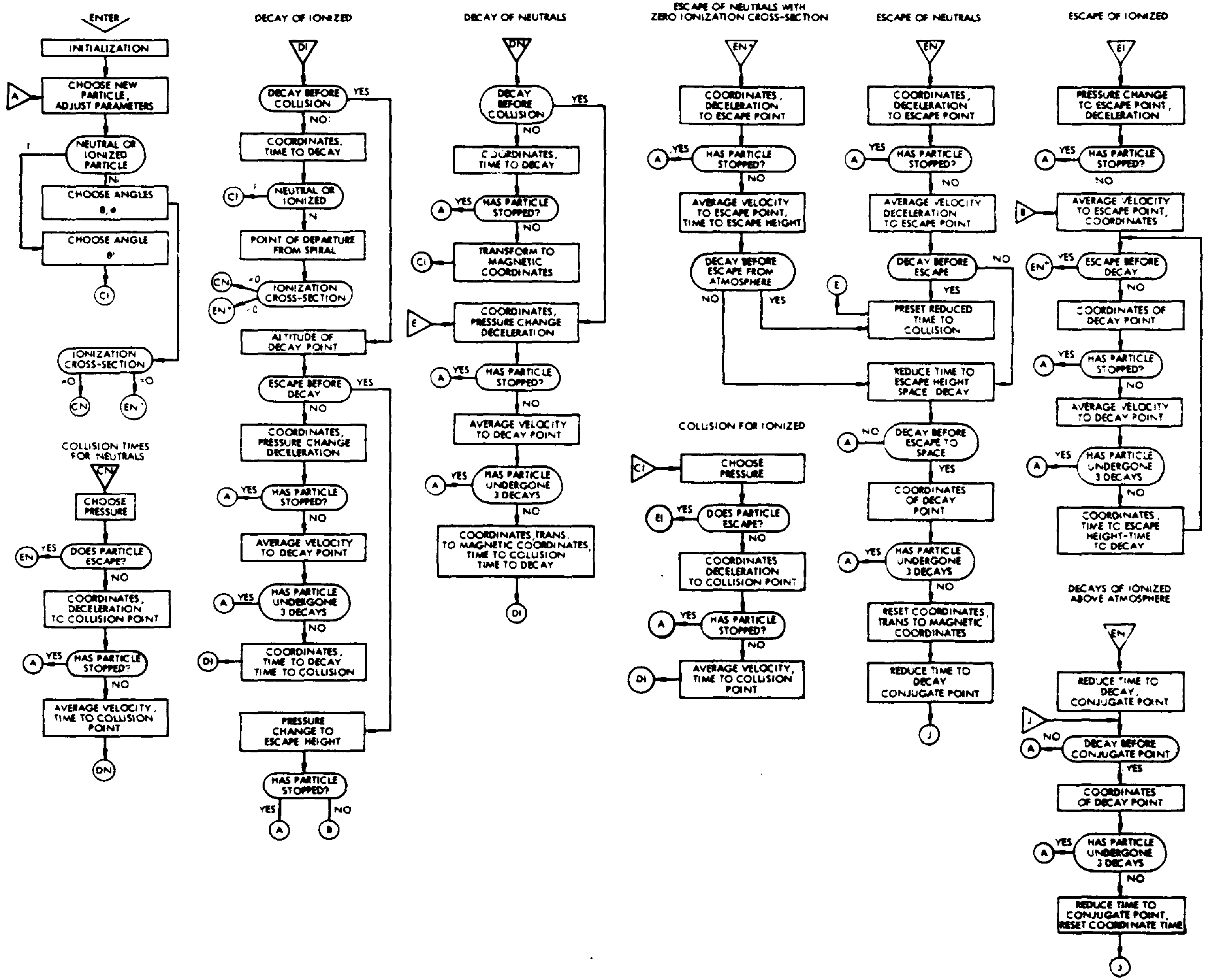


Fig. 7 Master Flow Chart

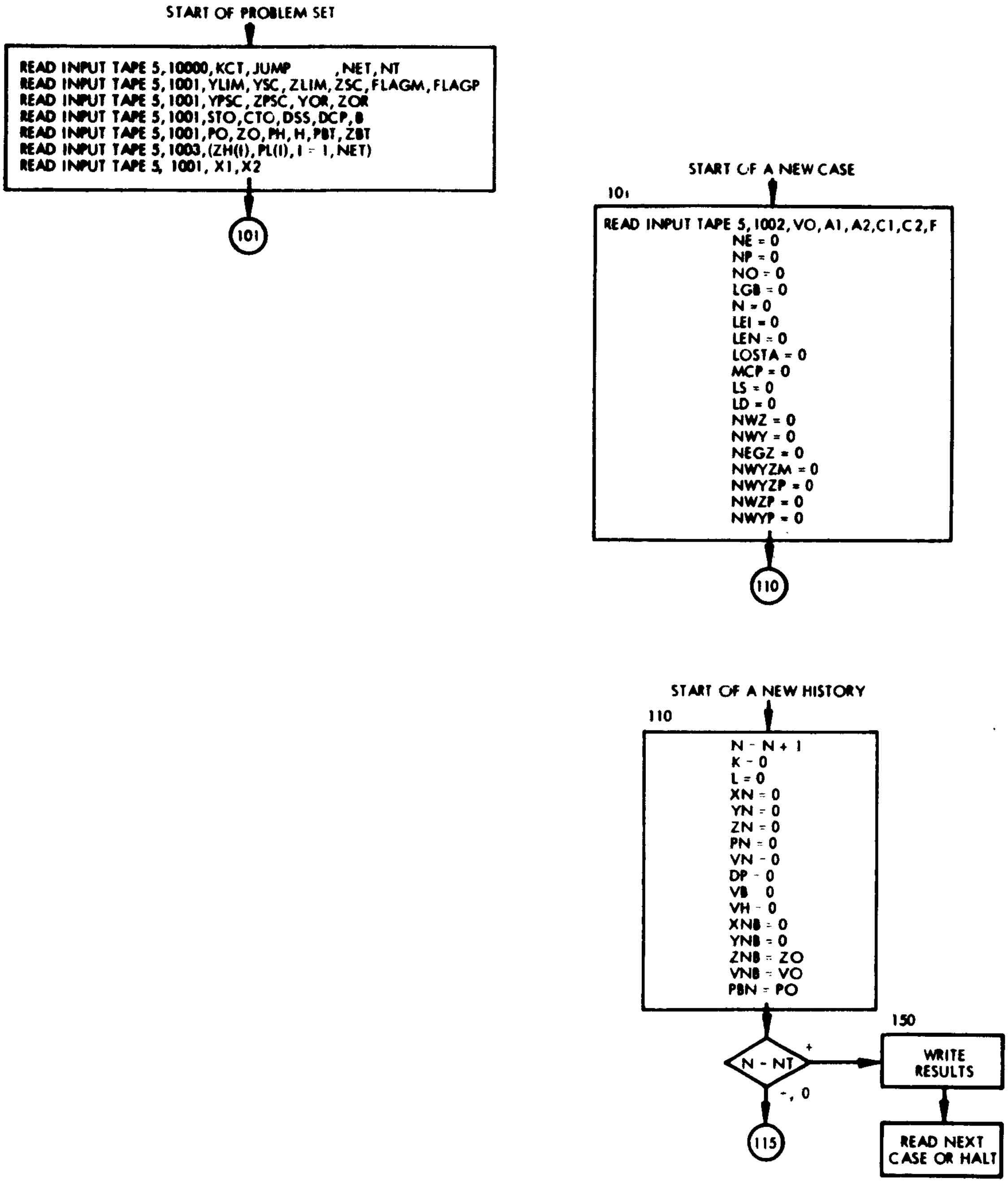


Fig. 8 Initialization Flow Chart

CALCULATION OF DECAY TIMES

Eqs. (3.28, 29, and 30)

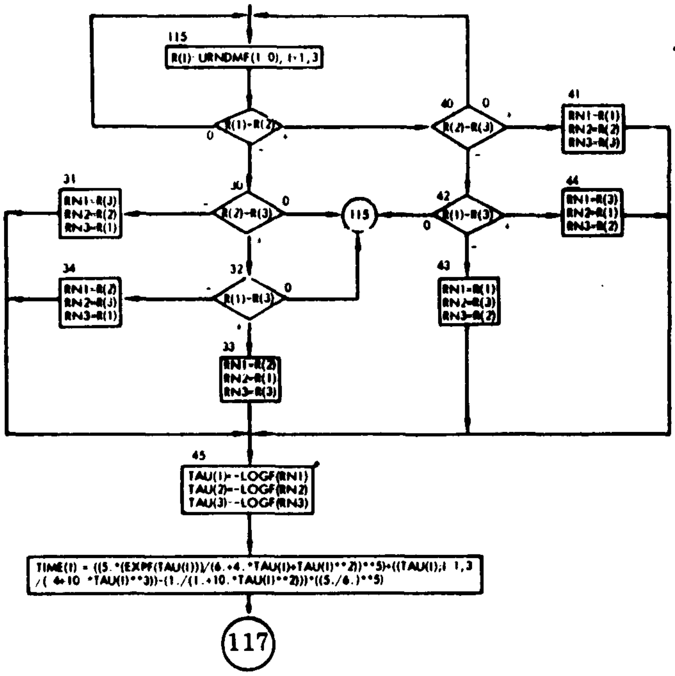


Fig. 9a Decay Times Flow Chart (used in production runs for Ref. 10)

CALCULATION OF DECAY TIMES
(Eqs. (3.18, 19, and 30)

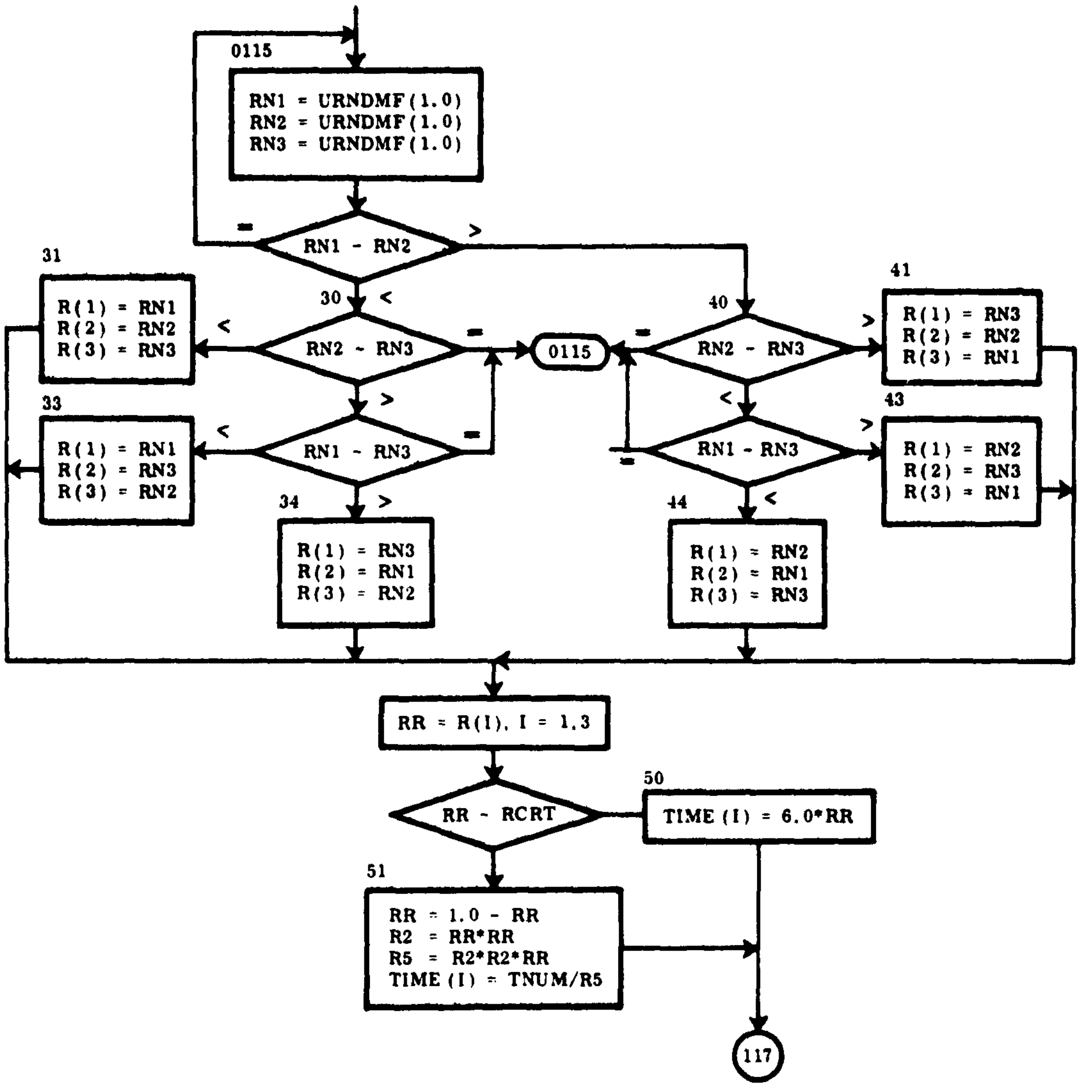


Fig. 9b Decay Times Flow Chart Yielding Correct Late Time Effects

CALCULATION OF REDUCED TIMES AND INITIAL CONDITIONS

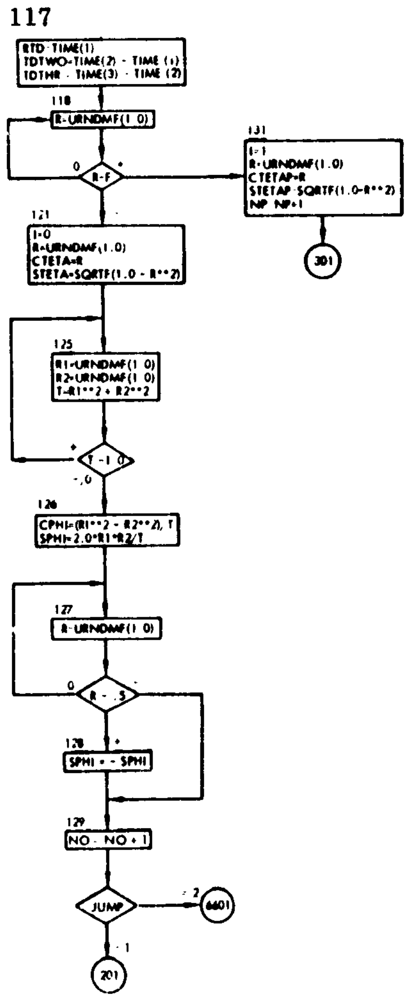


Fig. 9c Decay Times Flow Chart (completion)

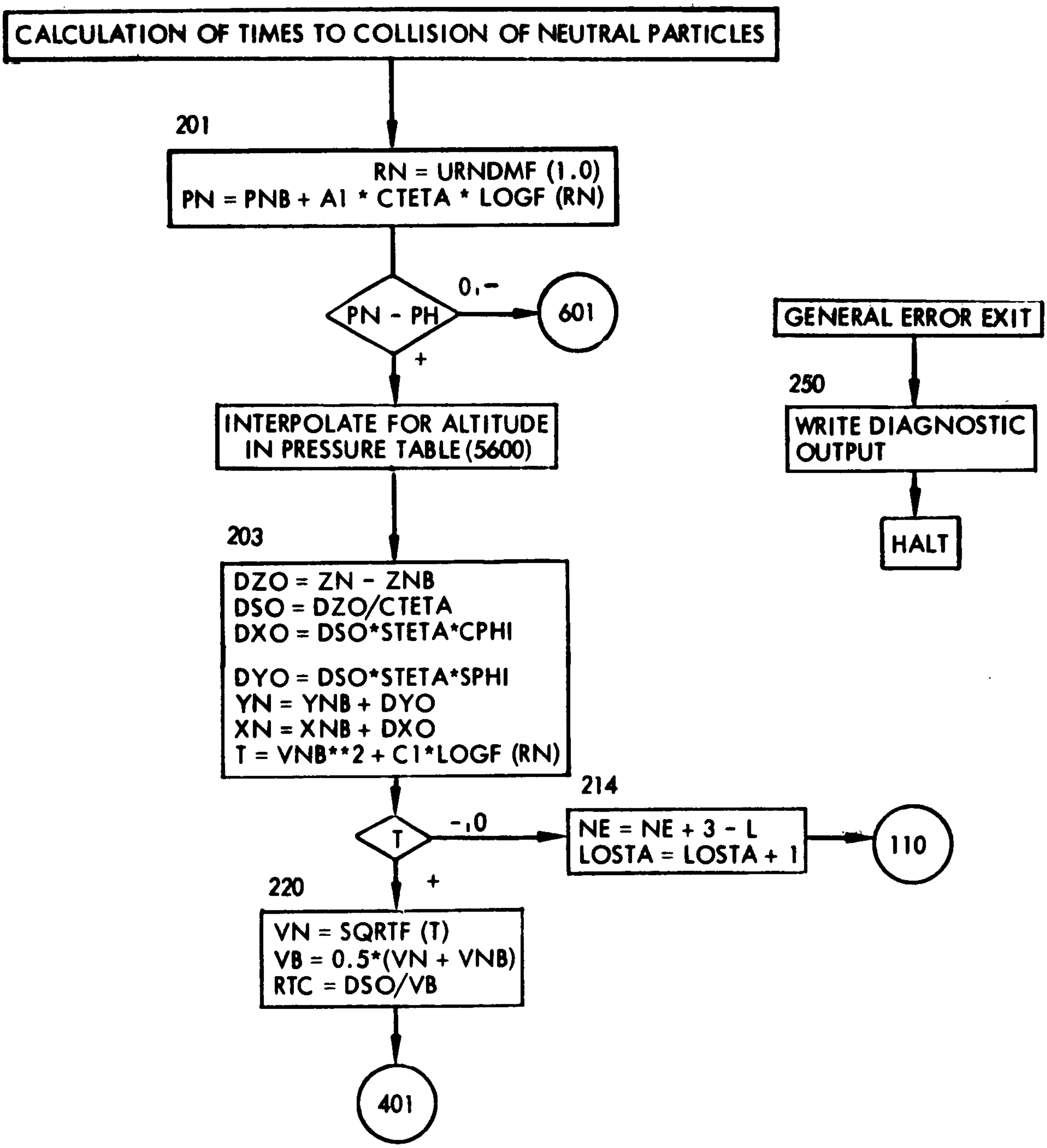


Fig. 10 Neutral Collision Flow Chart

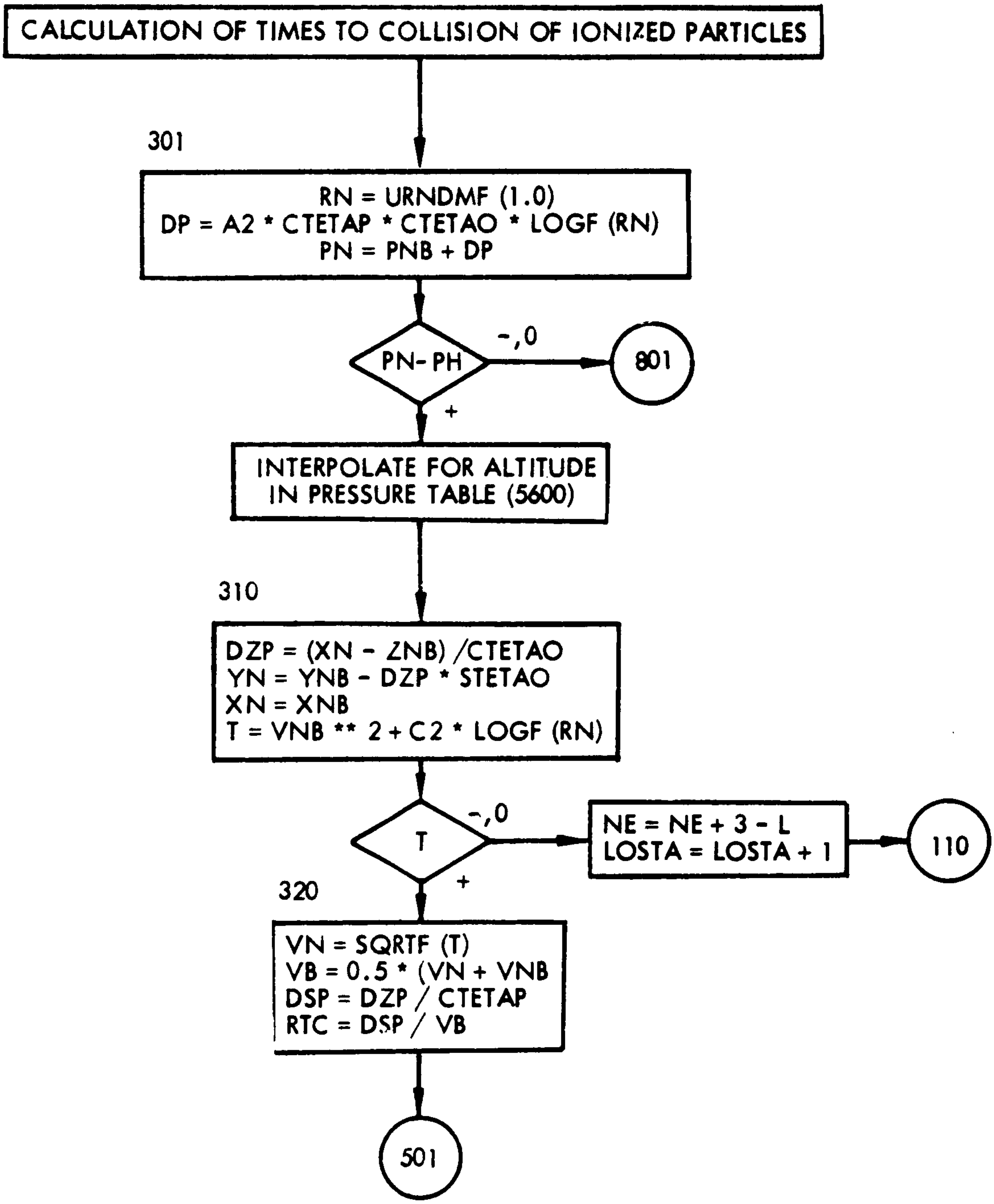


Fig. 11 Ionized Collision Flow Chart

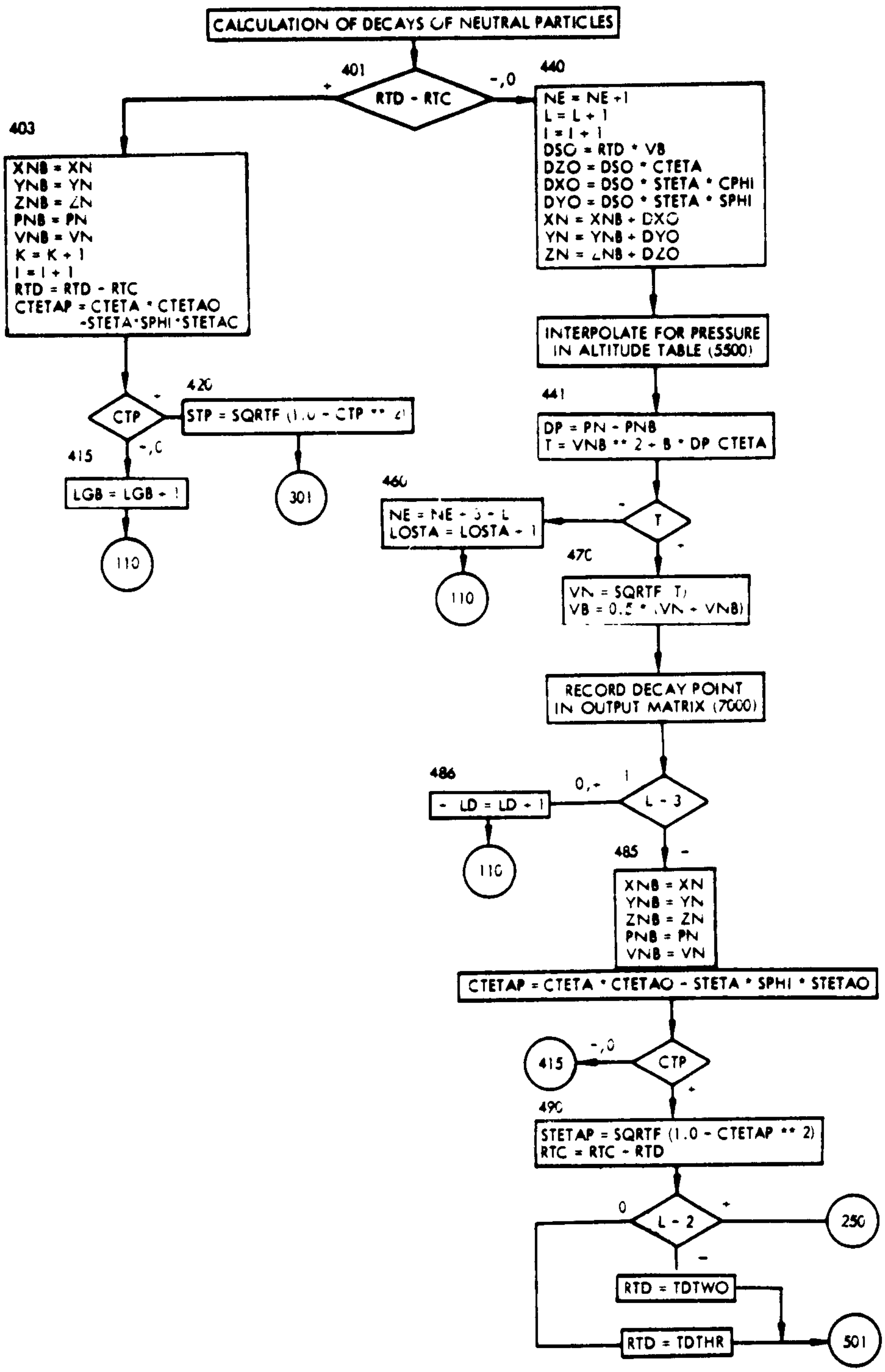


Fig. 12 Neutral Decay Flow Chart

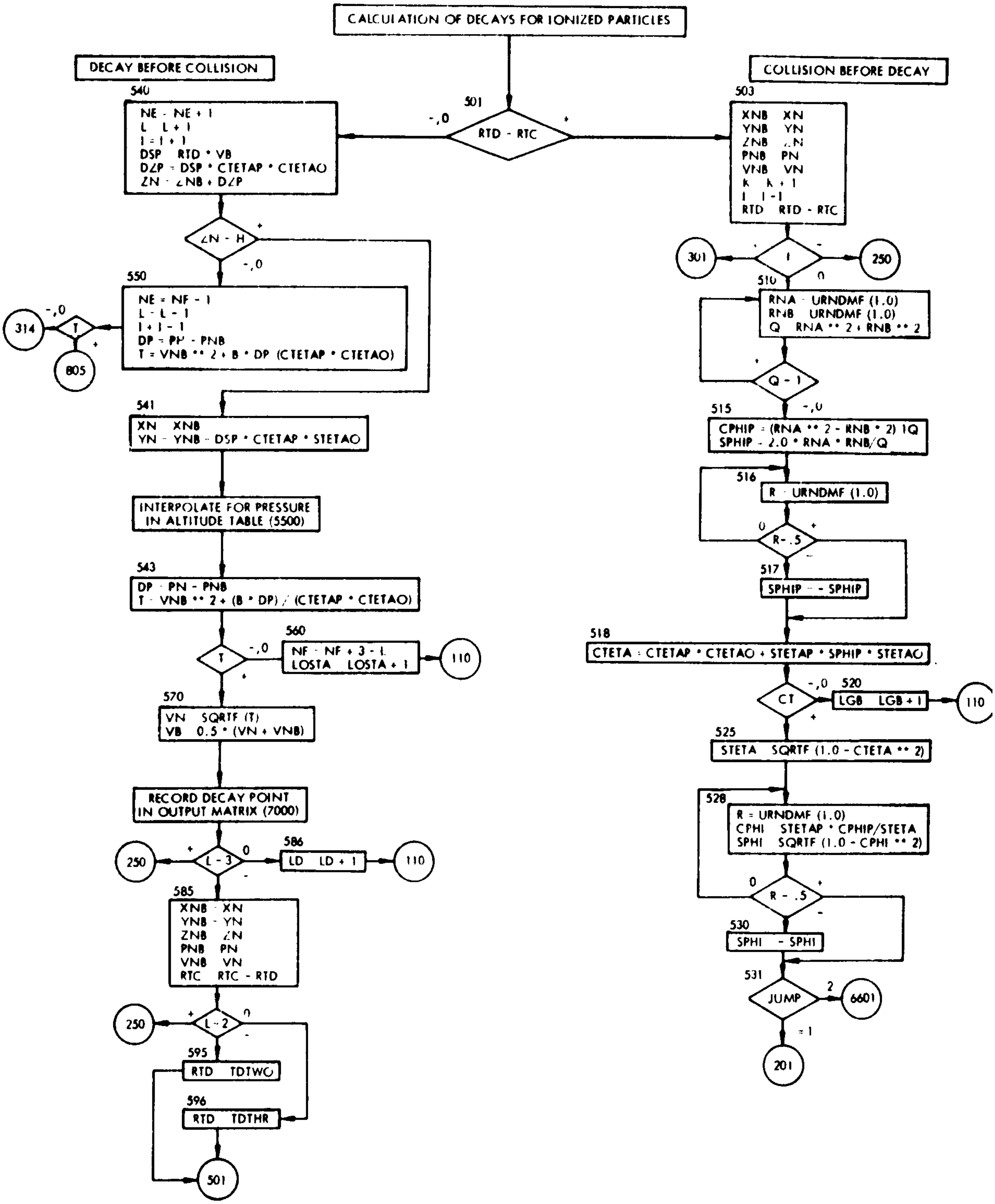


Fig. 13 Ionized Decay Flow Chart

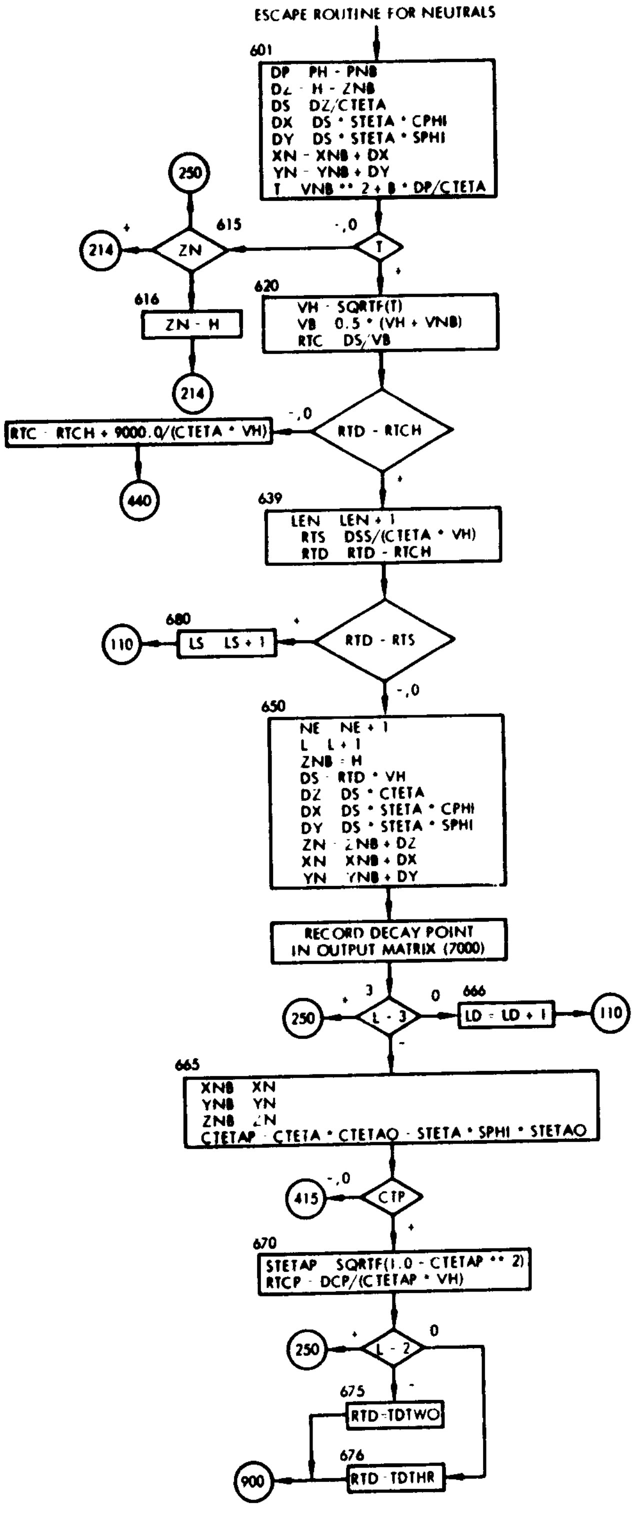


Fig. 14 Neutral Escape Flow Chart

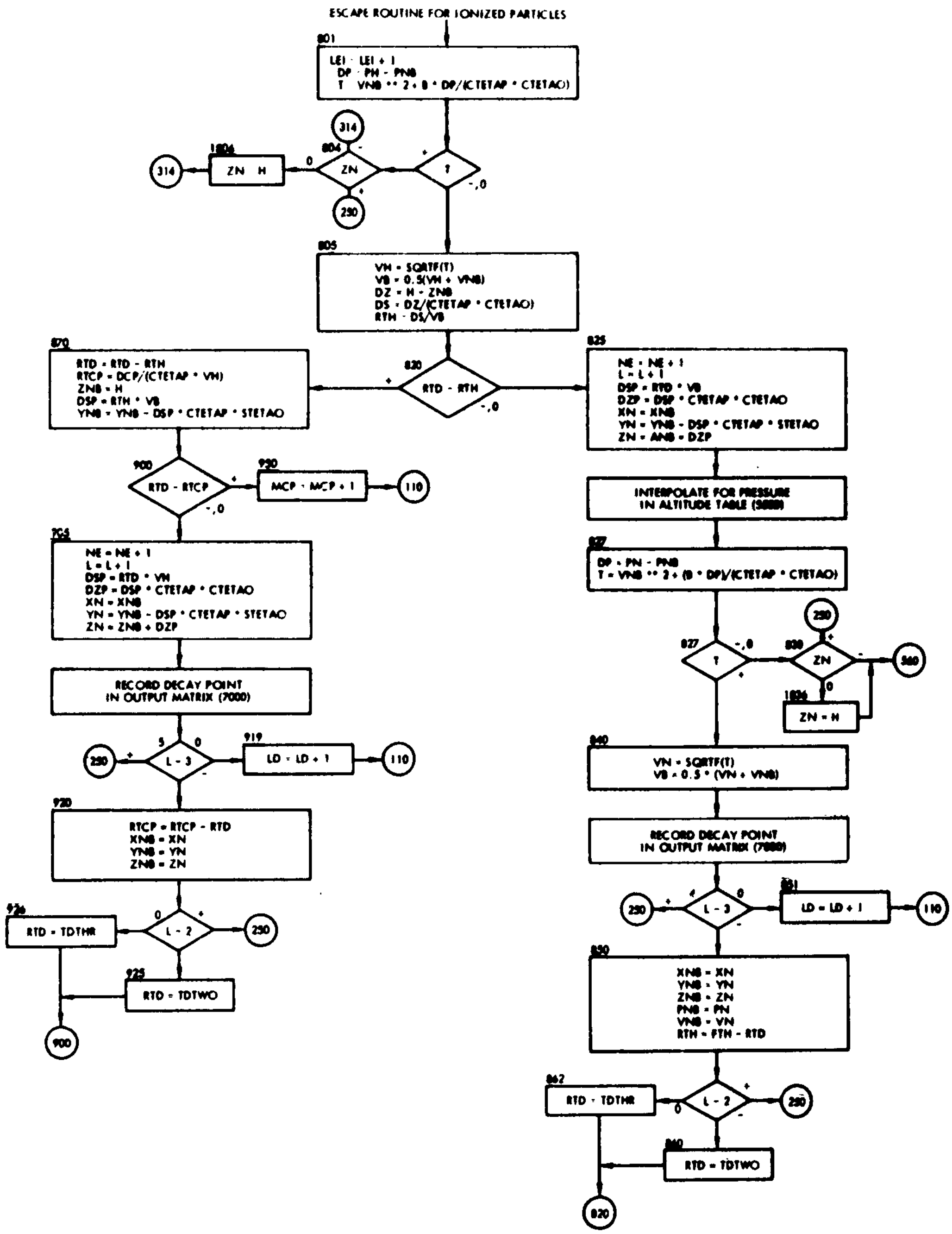


Fig. 15 Ionized Escape Flow Chart

ESCAPE ROUTINE FOR NEUTRALS WITH ZERO IONIZATION CROSS-SECTION

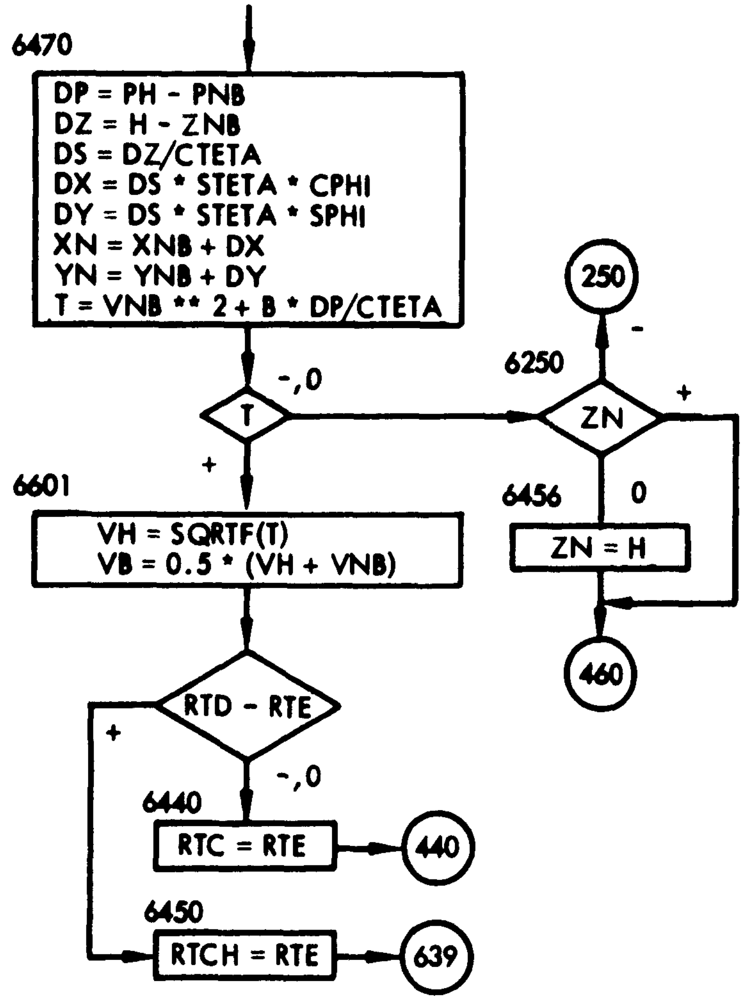


Fig. 16 Neutral Escape (σ_ℓ = 0) Flow Chart

Section 5

FORTRAN LISTING AND SAMPLE PROBLEM

This is the final version of the code using Eqs. (3.18), (3.19), and (3.20) for the decay times. This version gives correct late time effects.

```
*     FORTRAN
C     207-85    ORMONDE  X45343          DEMO FINAL VERSION
C   CHANGED PROGRAM TO USE TWO SIGMAS FOR EACH CASE W/COLUMN ONE OF NEGA-
C    TIVE MATRIX NONE ZERO               AUGUST 14, 1962
C      DEMO WITH BOTH PLOT AND MATRIX  5-31-62     BACKWARD TEST IS IN
C       MONTE  CARLO   FOR   DEMO    MAIN  PROGRAM  ORMONDE 52-10 X45219
C       NEW NAME   OLD NAME              DESCRIPTION
C       KCT        KOUNTY                TOTAL CARD COUNT
C       JUMP                             INDICATOR-CROSS-SECTION
C       NO                               NUMBER OF NEUTRAL PARTICLES  (N ZERO)
C       NP                               NUMBER OF IONIZED PARTICLES  (N+)
C       FRI        F                     FRACTION NEUTRAL
C       NET        N6                    NUMBER OF ENTRIES IN TABLE
C       NTP        NT                    TOTAL NUMBER OF PARITCLES
C       KCS        KOUNT                 CARD COUNT FLAG
C       CTO        CTETAO                COSINE THETA  ZERO
C       STO        STETAO                SIN THETA ZERO
C       CTP        CTETAP                COSINE THETA PRIME
C       STP        STETAP                SIN THETA PRIME
C       CT         CTETA                 COSINE THETA
C       ST         STETA                 SIN THETA
C       CP         CPHI                  COSIN PHI
C       SP         SPHI                  SIN PHI
C       CPP        CPHIP                 COSINE PHI PRIME
C       SPP        SPHIP                 SIN PHI PRIME
C
C
C
C
        DIMENSION T(3)
      DIMENSION R(3),TAU(3),TIME(3),ZH(50),PL(50),S(3)
      DIMENSION MP(19,50),MN(19,50)
      DIMENSION ISUMMP(19),ISUMMN(19),IROW(50)
      EQUIVALENCE (KCT,KOUNTY),(FRI,F),          (KCS,KOUNT),(CTETAO,CTO)
     1,(STETAO,STO),(CTETAP,CTP),(STETAP,STP),(CTETA,CT),(STETA,ST),(SPH
     2I,SP),(CPHI,CP),(CPHIP,CPP),(SPHIP,SPP)
1000    FORMAT (12I6)
1001  FORMAT (6F12.0)
1002    FORMAT (6E12.8)
1003    FORMAT (2E12.8)
1007  FORMAT(1H ,8I8)
1008  FORMAT(1H 4X,3HLGB5X,3HLEN5X,3HLEI5X,3HMCP6X,2HLS5X,2HLD6X,5HLOSTA)
     1)
1010  FORMAT (6H1  NO=I6,5H  NP=I6,6H  NWY=I6,6H  NWZ=I6,7H  NWYP=I6,
     17H  NWZP=I6,8H  NWYZM=I6,8H  NWYZP=I6,7H  NEGZ=I6)
1011  FORMAT(1H 7X,2HVO14X,2HPH15X,1HH15X,2HA115X,2HA215X,2HC115X,2HC2)
1012  FORMAT (6E16.8,I5)
1013  FORMAT(1H 7X,2HZO14X,4HYLIM11X,3HYSC12X,4HZLIM11X,3HZSC15X,1HB10X,
     14HJUMP)
1014    FORMAT(5E17.8)
1015  FORMAT(7E16.8)
1016  FORMAT(1H0)
1021    FORMAT    (40H0   HISTOGRAM FOR POSITIVE VALUES OF Y    )
```

```
1022  FORMAT(1H ,19I6)
1023   FORMAT    (40H1   HISTOGRAM FOR NEGATIVE VALUES OF Y     )
       READ INPUT TAPE 5,1000,KCT,JUMP    ,NET,NT
      READ  INPUT  TAPE  5,1001,YLIM,YSC,ZLIM,ZSC,FLAGM,FLAGP
      READ  INPUT  TAPE  5,1001,YPSC,ZPSC,YOR,ZOR
       READ  INPUT  TAPE  5,1001,STO,CTO,DSS,DCP,B
       READ  INPUT  TAPE  5,1001,PO,ZO,PH,H,PBT,ZBT
       READ  INPUT  TAPE  5,1003,(ZH(I),PL(I),I=1,NET)
      READ  INPUT  TAPE  5,  1001,  X1,X2
      DUMMY=INTRMF(X1,X2)
      KOUNT=0
         REWIND 27
      RYPSC=1.0/YPSC
      RZPSC=1.0/ZPSC
      TNUM=(5./6.)**5
      RCRT=1./6.
C
C
C
C
C
C
C
C
C
C
C
C***********************************************************************



C*************************START     OF    A   NEW     CASE*******************
C***********************************************************************
101   READ  INPUT  TAPE  5,1002,VO,A1,A2,C1,C2,F
                    NE=0
                    NP=0
                    NO=0
                    LGB=0
                    N=0
                    LEI=0
                    LEN=0
                    LOSTA=0
                   MCP=0
                    LS=0
      LD  =  0
                   NWZ=0
                   NWY=0
              NEGZ=0
              NWYZM=0
              NWYZP=0
              NWZP=0
              NWYP=0
C***********************************************************************
C****************START   OF   A   NEW   HISTORY****************************
C***********************************************************************
110                          N=N+1
                             K=0
                             L=0
                             XN=0
```

```
                                  YN=0
                                  ZN=0
                                  PN=0
                                  VN=0
                                  DP=0
                                  VB=0
                                  VH=0
                                  XNB=0
                                  YNB=0
                                  ZNB=ZO
                                  VNB=VO
                                  PNB=PO
C
C***************HAVE NT PARTICLES BEEN  PROCESSED*************************
C
      IF(N-NT)0115,0115,0150
C************************************************************************
C*************************END      OF      CASE***************************
C************************************************************************
150   CONTINUE
      DO 152 IY=1,19
      ISUMMP(IY)=0
      ISUMMN(IY)=0
      DO 152 IZ=5,40
      ISUMMP(IY)=MP(IY,IZ)+ISUMMP(IY)
      ISUMMN(IY)=MN(IY,IZ)+ISUMMN(IY)
  152 CONTINUE
      DO 155 IZ=1,50
      IROW(IZ)=0
      DO 155 IY=1,19
      IROW(IZ)=IROW(IZ)+MP(IY,IZ)+MN(IY,IZ)
  155 CONTINUE
      WRITE OUTPUT TAPE 6,1010,NC,NP,NWY,NWZ,NWYP,NWZP,NWYZM,NWYZP,NEGZ
      WRITE OUTPUT TAPE  6,1008
      WRITE OUTPUT TAPE 6,1007,LGB,LEN,LEI,MCP,LS,LD,LOSTA
              WRITE OUTPUT TAPE 6,1011
      WRITE OUTPUT TAPE 6,1015,VO,PH,H,A1,A2,C1,C2
              WRITE OUTPUT TAPE 6,1013
      WRITE OUTPUT TAPE 6,1012,ZO,YLIM,YSC,ZLIM,ZSC,B,JUMP
              WRITE OUTPUT TAPE 6,1021
              WRITE OUTPUT TAPE 6,1022,MP
              WRITE OUTPUT TAPE 6,1023
              WRITE OUTPUT TAPE 6,1022,MN
      WRITE OUTPUT TAPE 6, 1016
      WRITE OUTPUT TAPE 6, 1022, ISUMMP
      WRITE OUTPUT TAPE 6, 1022, ISUMMN
      WRITE OUTPUT TAPE 6, 1022, IROW
              DO 8000 I=1,19
              DO 8000 J=1,50
              MN(I,J)=0
              MP(I,J)=0
 8000         CONTINUE
         END FILE 27
```

```
              YN=0
              ZN=0
              PN=0
              VN=0
              DP=0
              VB=0
              VH=0
              XNB=0
              YNB=0
              ZNB=ZO
              VNB=VO
              PNB=PO
C
C****************HAVE NT PARTICLES BEEN  PROCESSED*************************
C
      IF(N-NT)0115,0115,0150
C*************************************************************************
C***********************END      OF      CASE******************************
C*************************************************************************
150   CONTINUE
      DO 152 IY=1,19
      ISUMMP(IY)=0
      ISUMMN(IY)=0
      DO 152 IZ=5,40
      ISUMMP(IY)=MP(IY,IZ)+ISUMMP(IY)
      ISUMMN(IY)=MN(IY,IZ)+ISUMMN(IY)
  152 CONTINUE
      DO 155 IZ=1,50
      IROW(IZ)=0
      DO 155 IY=1,19
      IROW(IZ)=IROW(IZ)+MP(IY,IZ)+MN(IY,IZ)
  155 CONTINUE
      WRITE OUTPUT TAPE 6,1010,NC,NP,NWY,NWZ,NWYP,NWZP,NWYZM,NWYZP,NEGZ
      WRITE OUTPUT TAPE  6,1008
      WRITE OUTPUT TAPE 6,1007,LGB,LEN,LEI,MCP,LS,LD,LOSTA
              WRITE OUTPUT TAPE 6,1011
      WRITE OUTPUT TAPE 6,1015,VO,PH,H,A1,A2,C1,C2
              WRITE OUTPUT TAPE 6,1013
      WRITE OUTPUT TAPE 6,1012,ZO,YLIM,YSC,ZLIM,ZSC,B,JUMP
              WRITE OUTPUT TAPE 6,1021
              WRITE OUTPUT TAPE 6,1022,MP
              WRITE OUTPUT TAPE 6,1023
              WRITE OUTPUT TAPE 6,1022,MN
      WRITE OUTPUT TAPE 6, 1016
      WRITE OUTPUT TAPE 6, 1022, ISUMMP
      WRITE OUTPUT TAPE 6, 1022, ISUMMN
      WRITE OUTPUT TAPE 6, 1022, IROW
              DO 8000 I=1,19
              DO 8000 J=1,50
              MN(I,J)=0
              MP(I,J)=0
8000          CONTINUE
         END FILE 27
```

```
                 KOUNT=KOUNT+1
      IF (KOUNT-KOUNTY)101,151,151
 0151            JUMP=2
      GO TO 0101
C
C
C
C
C
C
C
C
C
C
C
C****************************************************************************
C*********************CALCULATION    OF    DECAY    TIMES*****************
C****************************************************************************
0115  RN1=URNDMF(1.0)
      RN2=URNDMF(1.0)
      RN3=URNDMF(1.0)
      IF(RN1-RN2)30,0115,40
30    IF(RN2-RN3)31,0115,32
31    R(1)=RN1
      R(2)=RN2
      R(3)=RN3
      GO TO 45
32    IF(RN1-RN3)33,0115,34
33    R(1)=RN1
      R(2)=RN3
      R(3)=RN2
      GO TO 45
34    R(1)=RN3
      R(2)=RN1
      R(3)=RN2
      GO TO 45
40    IF(RN2-RN3)42,0115,41
41    R(1)=RN3
      R(2)=RN2
      R(3)=RN1
      GO TO 45
42    IF(RN1-RN3)44,0115,43
43    R(1)=RN2
      R(2)=RN3
      R(3)=RN1
      GO TO 45
44    R(1)=RN2
      R(2)=RN1
      R(3)=RN3
45    DO 0117 I=1,3
      RR=R(I)
      IF(RR-RCRT) 50,50,51
50    TIME(I)=6.*RR
      GO TO 0117
51    RR=1.-RR
      R2=RR*RR
      R5=R2*R2*RR
      TIME(I)=TNUM/R5
0117  CONTINUE
```

```
                  RTD=TIME(1)
                  TDTWO=TIME(2)-TIME(1)
                  TDTHR=TIME(3)-TIME(2)
 0120             CONTINUE
C**********CHOOSE INITIAL DIRECTION OF MOTION AND TYPE OF PARTICLE**********
 0118             R=URNDMF(1.0)
      IF(R-F)0121,0118,0131
 0121             I=0
 0122             R=URNDMF(1.0)
                  CTETA=R
                  STETA=SQRTF(1.0-R**2)
 0125             R1=URNDMF(1.0)
                  R2=URNDMF(1.0)
                  T=R1**2+R2**2
      IF(T-1.0)0126,0126,0125
 0126             CPHI=(R1**2-R2**2)/T
                  SPHI=2.0*R1*R2/T
 0127             R=URNDMF(1.0)
      IF(R-0.5)0129,0127,0128
 0128             SPHI=-SPHI
 0129             NO=NO+1
      GO TO (0201,6601),JUMP



 0131             I=1
 0132             R=URNDMF(1.0)
                  CTETAP=R
                  STETAP=SQRTF(1.0-R**2)
                  NP=NP+1
      GO TO 0301
C
C
C
C
C
C
C
C
C
C*************************************************************************
C******CALCULATION  OF  TIMES  TO  COLLISION  OF NEUTRAL PARTICLES*********
C*************************************************************************
 0201             RN=URNDMF(1.0)
      PN=PNB+A1*CTETA*LOGF(RN)
C************** DOES PARTICLE ESCAPE WITHOUT COLLISION*********************
      IF(PN-PH)0601,0601,204
204   IF(PN-PBT) 202,110,110
202      ASSIGN 203 TO IH
         GO TO 5600
203               DZO=ZN-ZNB
                  DSO=DZO/CTETA
 0208             DXO=DSO*STETA*CPHI
```

```
                DYO=DSO*STETA*SPHI
                YN=YNB+DYO
                XN=XNB+DXO
      T=VNB**2+C1*LOGF(RN)
      IF (T)0214,0214,0220
 0214           NF=NF+3-L
               LOSTA=LOSTA+1
C******PARTICLE SLOWED DOWN TO REST.END HISTORY.SIMILARLY AT OTHER POINTS**
      GO TO 0110
 0220           VN=SQRTF(T)
                VB=0.5*(VN+VNB)
                RTC=DSO/VB
      GO TO 0401
 0250           WRITE OUTPUT TAPE 6,0251,K,L,I
 0251           FORMAT(3I10)
                CALL DUMP
C*************************************************************************
C****************CALCULATION OF DECAYS OF NEUTRAL PARTICLES***************
C*************************************************************************
 0401           CONTINUE
      IF (RTD-RTC)0440,0440,0403
C****************************COLLISION BEFORE DECAY**************************
 0403           XNB=XN
                YNB=YN
                ZNB=ZN
                PNB=PN
                VNB=VN
                K=K+1
                I=I+1
                RTD=RTD-RTC
                CTETAP=CTETA*CTETAO-STETA*SPHI*STETAO
      IF(CTP)0415,0415,0420
 0415 LGB=LGB+1
      GO TO 0110
C*******END HISTORY. SIMILARLY AT ALL OTHER POINTS OF PROGRAM*************
 0420           STETAP=SQRTF(1.0-CTETAP**2)
      GO TO 0301
C**************************DECAY BEFORE COLLISION**************************
 0440           NF=NF+1
                L=L+1
                I=I+1
                DSO=RTD*VB
                DZO=DSO*CTETA
                DXO=DSO*STETA*CPHI
                DYO=DSO*STETA*SPHI
                XN=XNB+DXO
                YN=YNB+DYO
                ZN=ZNB+DZO
      IF(ZN-ZBT) 110,110,439
439   ASSIGN 441 TO IP
        GO TO 5500
441             DP=PN-PNB
                T=VNB**2+B*DP/CTETA
```

```
      IF (T)0460,0460,0470
 0460           NE=NE+3-L
               LOSTA=LOSTA+1
      GO TO 0110
 0470           VN=SQRTF(T)
                VB=0.5*(VN+VNB)
       ASSIGN 1 TO MCALL
       GO TO 7000
 1     CONTINUE
      IF(L-3)0485,0486,0250
 0486           LD=LD+1
      GO TO 0110
 0485           XNB=XN
                YNB=YN
                ZNB=ZN
                PNB=PN
                VNB=VN
                CTETAP=CTETA*CTETAO-STETA*SPHI*STETAO
      IF(CTP)0415,0415,0490
 0490           STETAP=SQRTF(1.0-CTETAP**2)
 0491           RTC=RTC-RTD
      IF (L-2)0495,0496,0250
 0495           RTD=TDTWO
      GO TO 0501
 0496           RTD=TDTHR
      GO TO 0501
C***********************************************************************
C*********CALCULATION OF TIMES TO COLLISION OF IONISED PARICLES**********
C***********************************************************************
 0301           RN=URNDMF(1.0)
      DP=A2*CTETAP*CTETAO*LOGF(RN)
                PN=PNB+DP
C************* DOES PARTICLE ESCAPE WITHOUT COLLISION*******************
      IF(PN-PH)0801,0801,0307
 0307 IF(PN-PBT)308,110,110
308       ASSIGN 310 TO IH
          GO TO 5600
310             DZP=(ZN-ZNB)/CTETAO
                YN=YNB-DZP*STETAO
                XN=XNB
      T=VNB**2+C2*LOGF(RN)
      IF (T)0314,0314,0320
 0314           NE=NE+3-L
               LOSTA=LOSTA+1
      GO TO 0110
 0320           VN=SQRTF(T)
                VB=0.5*(VN+VNB)
                DSP=DZP/CTETAP
                RTC=DSP/VB
      GO TO 0501
C
C
C
C
C
```

```
C
C
C
C
C
C
C
C
C*****************************************************************************
C****************CALCULATION OF DECAYS FOR IONIZED PARTICLES*********
C*****************************************************************************
C
 0501             CONTINUE
      IF (RTD-RTC)0540,0540,0503
C***************************COLLISION BEFORE DECAY***************************
503               XNB=XN
                  YNB=YN
                  ZNB=ZN
                  PNB=PN
                  VNB=VN
                  K=K+1
                  I=I-1
                  RTD=RTD-RTC
      IF (I)0250,0510,0301
 0510             RNA=URNDMF(1.0)
                  RNB=URNDMF(1.0)
                  Q=RNA**2+RNB**2
      IF(Q-1.0)0515,0515,0510
 0515             CPHIP=(RNA**2-RNB**2)/Q
                  SPHIP=2.0*RNA*RNB/Q
 0516             R=URNDMF(1.0)
      IF(R-0.5)0517,0516,0518
 0517             SPHIP=-SPHIP
 0518             CTETA=CTETAP*CTETAO+STETAP*SPHIP*STETAO
      IF(CT)0520,0520,0525
 0520 LGB=LGB+1
      GO TO 011C
 0525             STETA=SQRTF(1.0-CTETA**2)
 0528             R=URNDMF(1.0)
                  CPHI=STETAP*CPHIP/STETA
                  SPHI=SQRTF(1.0-CPHI**2)
      IF(R-0.5)0530,0528,0531
 0530             SPHI=-SPHI
 0531 GO TO (0201,6601),JUMP
C***************************DECAY BEFORE COLLISION***************************
 0540             NE=NE+1
                  L=L+1
                  I=I+1
                  DSP=RTD*VB
                  DZP=DSP*CTETAP*CTETAO
                  ZN=ZNB+DZP
       IF(ZN-ZBT)110,110,549
549   IF(ZN-H)0541,0550,0550
550                CONTINUE
```

```
                  NE=NE-1
                  L=L-1
                  I=I-1
                  DP=PH-PNB
                  T=VNB**2+B*DP/(CTETAP*CTETAO)
      IF(T)0314,0314,0805
 0541             XN=XNB
                  YN=YNB-DSP*CTETAP*STETAO
542         ASSIGN 543 TO IP
            GO TO 5500
543               DP=PN-PNB
                  T=VNB**2+(B*DP)/(CTETAP*CTETAO)
      IF (T)0560,0560,0570
 0560             NE=NE+3-L
                 LOSTA=LOSTA+1
      GO TO 0110
 0570             VN=SQRTF(T)
                  VB=0.5*(VN+VNB)
        ASSIGN 2 TO MCALL
        GO TO 7000
  2     CONTINUE
      IF(L-3)0585,0586,0250
 0586             LD=LD+1
      GO TO 0110
 0585             XNB=XN
                  YNB=YN
                  ZNB=ZN
                  PNB=PN
                  VNB=VN
                  RTC=RTC-RTD
      IF (L-2)0595,0596,0250
 0595             RTD=TDTWO
      GO TO 0501
 0596             RTD=TDTHR
      GO TO 0501
C
C
C
C
C
C
C
C
C*****************************************************************************
C*****ESCAPE ROUTINE FOR NEUTRALS WITH ZERO IONISATION CROSS-SECTION*******
C*****************************************************************************
C
C
 6601             CONTINUE
                  DP=PH-PNB
                  DZ=H-ZNB
                  DS=DZ/CTETA
                  DX=DS*STETA*CPHI
```

```
                  DY=DS*STETA*SPHI
                  XN=XNB+DX
                  YN=YNB+DY
                  T=VNB**2+B*DP/CTETA
        IF(T)6250,6250,6470
 6250   IF(ZN)0250,6456,0460
 6456             ZN=H
        GO TO 0460
 6470             VH=SQRTF(T)
                  VB=0.5*(VH+VNB)
                  RTE=DS/VB
 6327   IF(RTD-RTE)6440,6440,6450
 6440             RTC=RTE
        GO TO 0440
 6450             RTCH=RTE
        GO TO 0639
C**************************************************************************
C************ ESCAPE ROUTINE FOR NEUTRALS *******************************
C**************************************************************************
 0601             CONTINUE
                  DP=PH-PNB
                  DZ=H-ZNB
                  DS=DZ/CTETA
                  DX=DS*STETA*CPHI
                  DY=DS*STETA*SPHI
                  XN=XNB+DX
                  YN=YNB+DY
                  T=VNB**2+B*DP/CTETA
        IF(T)0615,0615,0620
 0615   IF(ZN)0250,0616,0214
 0616             ZN=H
        GO TO 0214
 0620             VH=SQRTF(T)
                  VB=0.5*(VH+VNB)
                  RTCH=DS/VB
        GO TO 0630
 0630   IF(RTD-RTCH)0635,0635,0639
 0635             RTC=RTCH+9000.0/(CTETA*VH)
        GO TO 0440
639              LEN=LEN+1
 0640             RTS=DSS/(CTETA*VH)
                  RTD=RTD-RTCH
        IF (RTD-RTS)0650,0650,0680
 0650             NE=NE+1
                  L=L+1
                  ZNB=H
                  DS=RTD*VH
                  DZ=DS*CTETA
                  DX=DS*STETA*CPHI
                  DY=DS*STETA*SPHI
                  ZN=ZNB+DZ
                  XN=XNB+DX
                  YN=YNB+DY
         ASSIGN 3 TO MCALL
         GO TO 7000
```

```
  3      CONTINUE
        IF(L-3)0665,0666,0250
  0666                LD=LD+1
        GO TO 0110
  0665                XNB=XN
                      YNB=YN
                      ZNB=ZN
                      CTETAP=CTETA*CTETAO-STETA*SPHI*STETAC
        IF(CTP)0415,0415,0670
  0670                STETAP=SQRTF(1.0-CTETAP**2)
                      RTCP=DCP/(CTETAP*VH)
        IF (L-2)0675,0676,0250
  0675                RTD=TDTWO
        GO TO 0900
  0676                RTD=TDTHR
        GO TO 0900
  0680                LS=LS+1
  4600                 CONTINUE
        GO TO 0110
C*************************************************************************
C**************** ESCAPE ROUTINE FOR IONIZED PARTICLES ****************
C*************************************************************************
  0801                CONTINUE
                    LEI=LEI+1
                      DP=PH-PNB
                      T=VNB**2+B*DP/(CTETAP*CTETAO)
        IF(T)0804,0804,0805
  0804  IF(ZN)0250,1806,0314
  1806                ZN=H
        GO TO 0314
  0805                VH=SQRTF(T)
                      VB=0.5*(VH+VNB)
                      DZ=H-ZNB
                      DS=DZ/(CTETAP*CTETAO)
                      RTH=DS/VB
        GO TO 0820
  0820  IF (RTD-RTH)0825,0825,0870
  0825                NE=NE+1
                      L=L+1
                      DSP=RTD*VB
                      DZP=DSP*CTETAP*CTETAO
                      XN=XNB
                      YN=YNB-DSP*CTETAP*STETAO
                      ZN=ZNB+DZP
        IF(ZN-ZBT) 110,110,826
 826       ASSIGN 827 TO IP
           GO TO 5500
 827                  DP=PN-PNB
                      T=VNB**2+(B*DP)/(CTETAP*CTETAO)
        IF(T)0830,0830,0840
  0830  IF(ZN)0250,1836,0560
  1836                ZN=H
        GO TO 0560
  0840                VN=SQRTF(T)
```

```
                VB=0.5*(VN+VNB)
      ASSIGN 4 TO MCALL
      GO TO 7000
4     CONTINUE
     IF(L-3)0850,0851,0250
0851            LD=LD+1
     GO TO 0110
0850            XNB=XN
                YNB=YN
                ZNB=ZN
                PNB=PN
                VNB=VN
                RTH=RTH-RTD
     IF (L-2)0860,0862,0250
0860            RTD=TDTWO
     GO TO 0820
0862            RTD=TDTHR
     GO TO 0820
0870            RTD=RTD-RTH
                RTCP=DCP/(CTETAP*VH)
                ZNB=H
                DSP=RTH*VB
                YNB=YNB-DSP*CTETAP*STETAO
     GO TO 0900
0900 IF (RTD-RTCP)0905,0905,0950
0905            NE=NE+1
                L=L+1
                DSP=RTD*VH
                DZP=DSP*CTETAP*CTETAO
                XN=XNB
                YN=YNB-DSP*CTETAP*STETAO
                ZN=ZNB+DZP
      ASSIGN 5 TO MCALL
      GO TO 7000
5     CONTINUE
     IF(L-3)0920,0919,0250
0919            LD=LD+1
     GO TO 0110
0920            RTCP=RTCP-RTD
                XNB=XN
                YNB=YN
                ZNB=ZN
     IF (L-2)0925,0926,0250
0925            RTD=TDTWO
     GO TO 0900
0926            RTD=TDTHR
     GO TO 0900
950               CONTINUE
               MCP=MCP+1
4900              CONTINUE
     GO TO 0110
C
C***********************************************************************
```

```
C    ************ INTERPPOLATION  SUBROUTINE   FOR   PRESSSURE**********
C***********************************************************************
5500    IF (ZN-ZH(1)) 5520,5510,5501
 5501 IF(ZN-ZH(NET))5502,5515,5525
5502    DO 5504 I=1,NET
          IF (ZN-ZH(I)) 5503,5504,5504
5503    PN=((ZN-ZH(I-1))*PL(I)-(ZN-ZH(I))*PL(I-1))/(ZH(I)-ZH(I-1))
        PN=10.**PN
        GO TO 5599
5504    CONTINUE
        GO TO 5599
5510    PN=PL(1)
        PN=10.**PN
        GO TO 5599
5515    PN=PL(NET)
        PN=10.**PN
        GO TO 5599
5520   WRITE OUTPUT TAPE 6,5521,ZN
5521   FORMAT(1H 3X,9HALTITUDE=E16.8,21H BELOW RANGE OF TABLE)
        GO TO 5510
5525   WRITE OUTPUT TAPE 6,5526,ZN
5526   FORMAT(1H 3X,9HALTITUDE=E16.8,21H ABOVE RANGE OF TABLE)
        GO TO 5515
5599   GO TO IP,(441,543,827)
C***********************************************************************
C    ************ INTERPPOLATION  SUBROUTINE   FOR   ALTITUDES*************
C***********************************************************************
5600   X=LOGF(PN)*.43429448
5601       IF(X-PL(1)) 5602,5610,5620
5602       IF (X-PL(NET)) 5625,5615,5603
5603         DO 5605 I=1,NET
               IF(X-PL(I)) 5605,5605,5604
5604           ZN=((X-PL(I-1))*ZH(I)-(X-PL(I))*ZH(I-1))/(PL(I)-PL(I-1))
               GO TO 5699
5605         CONTINUE
               GO TO 5699
5610    ZN=ZH(1)
        GO TO 5699
5615    ZN=ZH(NET)
        GO TO 5699
5620   WRITE OUTPUT TAPE 6,5621,X
5621   FORMAT(1H 3X,6HLOG P=E16.8,21H ABOVE RANGE OF TABLE)
        GO TO 5610
5625   WRITE OUTPUT TAPE 6,5626,X
5626   FORMAT(1H 3X,6HLOG P=E16.8,21H BELOW RANGE OF TABLE)
        GO TO 5615
5699    GO TO IH,(203,310)
C
C
C
C
C
C
C***********************************************************************
C  **************PREPARATION   OF   OUTPUT   MATRIX*****************
C***********************************************************************
```

```
7000      CONTINUE
        IF (FLAGP) 7009,7020,7010
7009   YP=(YN*CTO)+((ZN-ZO)*STO)
       ZP=((ZN-ZO)*CTO)-(YN*STO)
        GO TO 7008
7010     YP=YN
         ZP=ZN
7008   IF(RYPSC-ABSF(YP))  7050,7050,7011
7011   IF(RZPSC-ABSF(ZP))  7060,7060,7012
7012   CONTINUE
7020   IF(FLAGM)7019,7999,7027
7019     YP=YN
         ZP=ZN
        GO TO 7021
7027     CONTINUE
       ZP=((ZN-ZO)*CTO)-(YN*STO)
       YP=(YN*CTO)+((ZN-ZO)*STO)
       IF(ZP) 7026,7026,7021
7026   NEGZ=NEGZ+1
       GO TO 7999
7021   IZ=(ZP/ZSC)+1.0
           IF(49-IZ) 7030,7022,7022
7022   IF(YP) 7120,7130,7130
7120   IY=-YP/YSC+1.0
       IF(18-XABSF(IY))7035,7125,7125


7125   MN(IY,IZ)=MN(IY,IZ)+1
       GO TO 7999
7130   IY=YP/YSC+1.0
       IF(18-XABSF(IY))7035,7132,7132
7132   MP(IY,IZ)=MP(IY,IZ)+1
       GO TO 7999
7030   NWZ=NWZ+1
           IF(19-XABSF(IY))7031,7999,7999
 7031    NWYZM=NWYZM+1
          GO   TO   7999
 7035   NWY=NWY+1
        IF(50-IZ) 7036,7999,7999
7036   NWYZM=NWYZM+1
         GO   TO   7999
7050   NWYP=NWYP+1
           IF(RZPSC-ABSF(ZP))  7051,7999,7999
 7051   NWYZP=NWYZP+1
        GO TO 7999
 7060   NWZP=NWZP+1
       GO TO 7999
 7999      CONTINUE
       GO TO MCALL,(1,2,3,4,5)
       END
*      DATA
```

List of Symbols

KCT	Total card count
JUMP	Indicator cross section
NET	Number of entries in table of atmosphere
NT	Total number of particles to be processed
Y LIM	Highest value of Y' in output matrix
YSC	Size of grid on Y' in output matrix
Z LIM	Highest value of Z' in output matrix
ZSC	Size of grid on Z' in output matrix
FLAGM	Indicator for matrix
FLAGP	Indicator for plot routine
YPSC	Y scale factor for plot routine
ZPSC	Z scale factor for plot routine
YOR	Y position of origin on plot
ZOR	Z position of origin on plot
STO	Sin θ_o
CTO	Cos ϑ_o
DSS	Distance beyond which particle is ignored
DCP	Distance to conjugate point
B	See Eq. (4.8)
P_o	Pressure at starting point
Z_o	Starting altitude
PH	Pressure at cut-off altitude
H	Cut-off altitude
PBT	Pressure at lower cut-off altitude
ZBT	Lower cut-off altitude
X_1 X_2	Initialization valves for the random number generator (IBM only)

STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE STATEMENTS

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
CPHIP	4439	10526	CPHI	4439	10527	CPP	4438	10526	CP	4439	10527	CTETAO	4446	10536
CTETAP	4444	10534	CTETA	4442	10532	CTO	4446	10536	CTP	4444	10534	CT	4442	10532
PHI	4448	10540	F	4448	10540	IROW	4283	10273	ISUMMN	4302	10316	ISUMMP	4321	10341
KCS	4447	10537	KCT	4449	10541	KCUNT	4447	10537	KCUNTY	4449	10541	MN	3283	06323
TP	4233	10211	PL	4374	10426	R	4433	10521	SPHIP	4437	10525	SPHI	4440	10530
SPP	4437	10525	SP	4440	10530	S	4324	10344	STETAO	4445	10535	STETAP	4443	10533
STETA	4441	10531	STO	4445	10535	STP	4443	10533	ST	4441	10531	TAU	4430	10516
TIME	4427	10513	T	4436	10524	ZH	4424	10510						

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN COMMON, DIMENSION, OR EQUIVALENCE STATEMENT

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
A1	2333	04435	A2	2332	04434	B	2331	04433	C1	2330	04432	C2	2329	04431
DCP	2328	04430	DP	2327	04427	DSC	2326	04426	DSP	2325	04425	DS	2324	04424
DSS	2323	04423	DUMMY	2322	04422	DXC	2321	04421	DX	2320	04420	DYO	2319	04417
DY	2318	04416	DZO	2317	04415	DZP	2316	04414	DZ	2315	04413	FLAGM	2314	04412
FLAGP	2313	04411	H	2312	04410	IH	2311	04407	IP	2310	04406	I	2309	04405
IY	2308	04404	IZ	2307	04403	JUMP	2306	04402	K	2305	04401	LD	2304	04400
LHI	2303	04377	LEN	2302	04376	LGB	2301	04375	LCSTA	2300	04374	L	2299	04373
LS	2298	04372	MCALL	2297	04371	MCP	2296	04370	NEGZ	2295	04367	NE	2294	04366
NET	2293	04365	NO	2292	04364	NP	2291	04363	N	2290	04362	NT	2289	04361
NWYP	2288	04360	NWY	2287	04357	NWYZM	2286	04356	NWYZP	2285	04355	NWZP	2284	04354
NWZ	2283	04353	PBT	2282	04352	PH	2281	04351	PNB	2280	04350	PN	2279	04347
PO	2278	04346	Q	2277	04345	R1	2276	04344	R2	2275	04343	R5	2274	04342
RCRT	2273	04341	RN1	2272	04340	RN2	2271	04337	RN3	2270	04336	RNA	2269	04335
RNB	2268	04334	RN	2267	04333	RR	2266	04332	RTCH	2265	04331	RTCP	2264	04330
RTC	2263	04327	RTO	2262	04326	RTE	2261	04325	RTH	2260	04324	RTS	2259	04323
RYPSC	2258	04322	RZPSC	2257	04321	TOTHR	2256	04320	TCTWO	2255	04317	TNLM	2254	04316
VR	2253	04315	VH	2252	04314	VNB	2251	04313	VN	2250	04312	VO	2249	04311
X1	2248	04310	X2	2247	04307	XNB	2246	04306	XN	2245	04305	X	2244	04304
YLIM	2243	04303	YNB	2242	04302	YN	2241	04301	YOR	2240	04300	YP	2239	04277
YPSC	2238	04276	YSC	2237	04275	ZBT	2236	04274	ZLIM	2235	04273	ZNB	2234	04272
ZN	2233	04271	ZOR	2232	04270	ZC	2231	04267	ZP	2230	04266	ZPSC	2229	04265
ZSC	2228	04264												

NO= 25000 NP= 0 NWY= 816 NWZ= 3805 NWYP= 0 NWZP= 0 NWYZM= 186 NWYZP= 0 NEGZ= 4441

LGB	LEN	LEI	MCP	LS	LD	LOSTA
4529	1717	13019	5322	101	14366	682

VC	PH	H	A1	A2	C1	C2
	0.48999999E-06	0.90000000E 03	0.23300000E-02	0.23300000E-02	0.35999999E 05	0.35999999E 05

ZC	YLIM	YSC	ZLIM	ZSC	B	JUMP
--	0.12500000E 04	0.50000000E 02	0.50000000E 04	0.09999999E 03	0.15499999E 08	2

HISTOGRAM FOR POSITIVE VALUES OF Y

5012	1640	1000	648	419	352	259	192	103	78	57	35	28	19	10	15	8	4	0
1697	998	735	513	420	300	205	153	94	75	47	33	33	20	22	9	6	8	0
1203	654	489	372	288	210	162	94	79	63	37	25	20	19	8	5	4	2	0
859	421	325	242	167	152	117	67	57	34	25	13	5	7	4	2	2	5	0
606	313	190	151	115	74	47	63	37	23	9	7	6	3	4	1	3	0	0
403	180	158	89	74	57	69	219	95	48	31	18	22	8	8	4	0	2	0
221	121	57	49	38	35	47	70	52	43	24	22	19	11	3	4	5	0	0
113	72	48	46	32	31	37	45	50	28	26	16	15	7	12	8	3	1	0
109	49	38	29	35	34	27	46	29	22	14	10	14	6	12	5	4	1	0
92	51	39	25	22	26	24	33	22	19	11	7	13	9	5	3	5	4	0
90	41	42	22	22	12	18	27	21	17	9	10	8	6	10	6	1	0	0
80	43	30	26	17	16	17	19	23	15	4	3	7	8	7	6	4	1	0
69	[illegible]	29	32	16	26	17	20	13	8	13	3	0	1	4	3	2	2	0
59	36	18	25	16	15	11	11	12	11	4	3	2	3	4	2	1	2	0
52	29	16	20	9	11	12	11	6	8	2	6	3	8	4	2	1	1	0
52	33	21	19	16	12	12	17	12	3	7	6	0	2	4	1	6	3	0
48	32	23	15	15	11	5	9	10	7	3	1	2	1	2	1	3	1	0
52	18	16	17	7	10	7	12	8	4	6	1	4	4	2	0	1	1	0
41	19	14	8	11	3	13	5	10	11	8	4	2	2	1	2	0	0	0
46	21	17	10	17	10	11	9	8	2	4	1	1	4	3	2	1	1	0
34	18	16	8	10	6	5	10	6	5	4	3	4	2	1	2	1	0	0
38	19	18	17	6	4	3	8	11	4	6	2	3	1	1	0	2	0	0
26	14	13	9	10	6	6	14	4	4	2	1	1	2	5	1	3	1	0
22	17	10	9	9	4	8	7	4	5	2	2	3	2	1	2	0	1	0
46	17	10	6	9	4	7	6	6	3	4	0	1	3	0	3	1	0	0
33	10	11	7	3	8	3	8	1	1	0	2	2	2	0	2	0	0	0
29	13	19	9	7	0	5	4	2	5	2	3	0	1	1	0	0	0	0
26	15	8	8	5	4	6	6	2	7	2	1	2	1	1	0	2	0	0
33	12	6	8	4	6	2	8	5	3	2	2	2	0	2	1	0	0	0
22	6	6	3	5	6	2	3	1	1	2	0	1	1	0	0	0	1	0
22	13	7	10	0	4	3	5	1	0	0	0	1	0	1	1	0	0	0
24	11	11	8	4	4	3	8	3	4	2	0	1	0	3	0	1	0	0
22	10	9	9	3	4	4	7		5	1	1	0	1	2	2	1	0	0
16	9	5	4	10	2	3	3	1	0	0	1	1	0	1	1	0	0	0
27	14	3	5	8	6	3	3	3	1	1	0	2	2	0	1	1	0	0
15	7	10	5	2	3	6	7	2	3	1	1	2	1	0	0	0	0	0
14	8	7	4	3	0	3	5	2	1	1	1	0	0	1	1	0	1	0
16	4	9	3	5	2	1	3	2	1	2	2	2	0	0	1	1	0	0
9	5	5	5	2	3	6	5	1	2	1	0	0	1	0	0	0	1	0
12	7	5	4	3	1	0	1	6	3	2	0	2	1	2	0	0	0	0
17	8	11	2	5	2	3	4	0	1	1	3	0	1	1	0	0	1	0
9	5	6	3	2	5	1	3	1	2	1	0	1	1	1	1	0	0	0
6	8	6	4	1	1	1	4	0	4	1	1	0	1	0	0	3	0	0
17	7	5	4	1	4	4	3	3	4	2	1	1	1	1	0	0	0	0
15	3	7	4	2	0	2	3	0	3	0	1	1	0	0	0	0	1	0
10	6	7	1	3	1	3	4	2	4	1	0	2	0	0	0	0	0	0
10	6	4	5	5	4	3	4	1	2	0	0	0	1	0	0	0	0	0
6	8	6	4	4	0	2	2	2	1	0	1	1	1	0	0	0	1	0
13	7	5	3	2	4	2	2	0	2	0	1	0	0	0	2	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

HISTOGRAM FOR NEGATIVE VALUES OF Y

	2844	402	44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1255	609	423	178	68	5	0	0	0	0	0	0	0	0	0	0	0	0	0
	970	461	352	257	178	99	34	0	0	0	0	0	0	0	0	0	0	0	0
	783	317	251	190	100	103	43	35	14	4	0	0	0	0	0	0	0	0	0
	524	262	160	127	78	56	46	24	21	19	14	9	0	1	0	0	0	0	0
	409	207	125	75	50	46	31	16	17	10	11	8	4	4	1	0	0	0	0
	339	130	94	77	37	38	24	15	11	17	12	4	2	5	3	8	4	0	0
	263	103	88	50	50	28	15	8	11	11	7	4	3	4	2	4	1	2	0
	161	84	74	44	28	24	12	7	10	12	7	7	2	7	9	0	2	0	0
	113	54	48	24	27	16	24	14	5	8	6	1	4	6	5	3	5	1	0
	90	47	33	17	27	7	15	6	3	6	6	3	1	1	5	2	0	2	0
	66	42	28	24	15	6	6	3	3	5	1	1	0	5	4	2	1	0	0
	52	34	23	12	15	9	7	3	2	3	1	0	1	0	2	0	0	0	0
	54	36	18	17	15	4	7	2	5	1	3	1	1	1	1	4	1	0	0
	51	18	12	11	5	9	4	3	1	3	1	0	1	1	2	1	2	0	0
	61	21	13	12	9	8	5	6	0	1	1	0	0	3	0	0	0	2	0
	41	15	13	12	7	8	7	8	1	2	0	0	1	1	1	1	0	1	0
	34	21	9	4	8	3	4	4	1	1	1	2	0	2	1	0	1	0	0
	41	14	9	7	5	7	3	0	2	2	1	1	0	0	0	0	1	0	0
	24	16	9	8	3	3	3	0	2	1	1	1	0	2	0	1	0	0	0
	31	10	12	8	2	5	3	1	0	3	1	1	1	1	0	0	1	0	0
	26	8	10	3	4	1	2	3	1	0	0	0	0	1	0	0	0	0	0
	27	10	7	0	5	3	2	2	3	0	0	0	0	0	2	0	0	0	0
	28	10	6	6	2	5	3	2	1	2	0	0	1	0	0	0	1	0	0
	23	9	7	7	3	4	1	0	0	0	1	2	0	0	0	1	0	0	0
	22	6	6	7	1	0	1	1	1	1	0	3	1	0	2	1	0	0	0
	21	8	12	3	5	3	2	0	2	0	1	0	0	0	0	0	0	0	0
	27	19	4	1	0	0	1	1	0	1	0	0	0	0	0	1	0	0	0
	10	11	12	2	0	0	2	1	0	0	0	1	0	0	0	0	0	0	0
	18	12	3	8	8	1	3	0	1	1	2	2	1	0	0	0	0	0	0
	18	6	5	6	6	1	1	1	1	2	0	0	0	0	2	0	0	0	0
	14	7	6	0	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0
	19	5	7	5	1	2	1	0	1	0	0	1	0	0	0	1	0	0	0
	19	6	4	4	9	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	11	9	6	3	2	2	1	0	0	0	0	0	0	0	1	0	0	0	0
	16	10	4	3	1	3	0	0	2	0	0	0	0	0	0	0	0	0	0
	11	15	6	2	0	1	1	0	1	0	1	2	0	0	0	1	0	0	0
	14	8	4	1	1	2	0	1	0	0	1	1	0	1	0	0	0	0	0
	14	6	3	1	4	1	1	0	1	0	0	0	0	1	0	0	0	0	0
	11	5	7	1	1	1	0	0	0	1	0	0	0	0	0	0	1	0	0
	7	3	5	3	1	1	0	2	1	0	0	0	0	0	0	0	0	0	0
	9	6	7	2	3	2	2	0	0	0	0	0	0	0	0	0	0	1	0
	9	3	4	2	1	2	3	0	0	0	0	0	0	1	0	1	0	1	0
	6	2	5	2	3	0	1	1	0	0	0	0	0	0	1	0	0	0	0
	6	7	4	2	3	2	4	1	0	0	1	1	0	0	0	0	0	0	0
	13	5	2	0	2	1	1	1	0	1	0	0	0	0	0	0	0	0	0
	10	5	3	4	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0
	8	9	4	1	0	4	0	0	0	1	1	0	1	0	0	1	0	0	0
	12	1	3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SUM POSITIVE COLUMNS	2619	1304	946	724	570	460	453	737	474	327	212	140	150	104	107	68	53	25	0
NEGATIVE COLUMNS	2703	1284	887	592	435	309	240	133	110	114	80	54	23	48	43	31	21	8	0
ROWS	13191	7906	6093	4344	2993	2492	1641	1244	974	774	633	528	449	406	326	368	308	268	249
	242	215	202	183	175	184	146	157	151	135	119	117	119	127	101	115	104	93	88
	78	77	83	74	68	79	73	70	70	69	63	0							

Section 6

ATMOSPHERIC PARAMETERS AS A FUNCTION OF ALTITUDE NEAR SUNSPOT MAXIMUM

altitude (km)	pressure (dyne/cm^2)
100	1.74×10^{-1}
120	3.4×10^{-2}
140	1.04×10^{-2}
160	5.1×10^{-3}
180	3.1×10^{-3}
200	1.95×10^{-3}
220	1.20×10^{-3}
240	8.5×10^{-4}
260	6.4×10^{-4}
280	4.7×10^{-4}
300	3.6×10^{-4}
320	2.7×10^{-4}
340	2.04×10^{-4}
360	1.54×10^{-4}
380	1.23×10^{-4}
400	9.8×10^{-5}
450	5.2×10^{-5}
500	2.9×10^{-5}
600	1.00×10^{-5}
700	3.5×10^{-6}
800	1.32×10^{-6}
900	4.9×10^{-7}
1000	1.90×10^{-7}
1200	3.2×10^{-8}
1400	6.7×10^{-9}
1600	2.1×10^{-9}
1800	1.14×10^{-9}
2000	9.5×10^{-10}
2500	7.2×10^{-10}

ATMOSPHERIC PARAMETERS AS A FUNCTION OF ALTITUDE NEAR SUNSPOT MINIMUM

altitude (km)	pressure (dyne/cm^2)
100	1.74×10^{-1}
120	2.1×10^{-2}
140	4.6×10^{-3}
160	1.86×10^{-3}
180	9.1×10^{-4}
200	5.0×10^{-4}
220	2.8×10^{-4}
240	1.77×10^{-4}
260	1.14×10^{-4}
280	7.6×10^{-5}
300	5.1×10^{-5}
320	3.5×10^{-5}
340	2.34×10^{-5}
360	1.66×10^{-5}
380	1.14×10^{-5}
400	8.3×10^{-6}
450	3.6×10^{-6}
500	1.66×10^{-6}
600	3.4×10^{-7}
700	7.9×10^{-8}
800	2.4×10^{-8}
900	1.26×10^{-8}
1000	9.8×10^{-9}
1200	7.2×10^{-9}
1400	6.2×10^{-9}
1600	5.1×10^{-9}
1800	4.3×10^{-9}
2000	3.8×10^{-9}
2500	2.8×10^{-9}

Section 7

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